

# **RMS DELAY SPREAD ESTIMATION FOR TROPICAL AREA DURING RAIN EVENT**

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## **ABSTRACT**

*A radio wave propagation in many Tropical area undergoes signal attenuation due to depth of the foliage, losses due to rain, seasonal variations etc. The propagation of Radio signals in vegetational area during rain event is an important issue which impacts on the present and future scope of the ultra-wideband technology. Hence in this paper a mathematical model is developed to estimate the RMS Delay spread characteristics of the channel for single reflections of signals for various rain fall rates. Applications The results of this investigation are discussed, covering density of the trees, foliage type in the presence of line-of-sight (LOS) and non-line-of-sight (NLOS) paths. Simulation results shows that the RMS Delay spread of the transmitted signal increases during a rain event (wet season) and it is observed that it decreases during no rain event (dry season).*

**Keywords:** ToA (Time of Arrival),pdf(probability density function) ,second central moment(CM). RMS (Root mean square), LTE (Long Term Evolution).

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## **1. INTRODUCTION**

The frequency reuse techniques in cellular communication had made the reduction in base station antenna heights has forced many Researchers to investigate the effect of vegetation on radiowave propagation in many tropical areas. These investigations will assist in Network planning for optimizing spectrum utilization and enhancing the quality of services provided by the Service Provider. The study of propagation through vegetation at UHF band during rain event is challenging due to variations in foliage depth, the distance between the BS(Base station) and the Subscriber Device(SD). Since it is well known that the rainfall affects

propagation only at high frequencies of above 5 GHz, little attention is given to the foliage medium that can potentially become an important source of absorption and attenuation of the propagating UHF (300 MHz-3 GHz) band [1].

In recent years, OFDM has advanced to achieve spectral efficiencies, while overcoming the inter symbol interference (ISI). Starting with 3G, the wireless communications standards have incorporated Orthogonal Frequency Division Multiplexing which mitigates the ISI in a rich multipath phenomenon. Cyclic prefix extension of the OFDM symbol neglects and overcome ISI if the cyclic prefix length is better than the maximum excess delay of the radio channel [2]. The ISI is directly correlated to the multipath propagation phenomenon resulting from various objects found in the environment. The time dispersion parameter namely the RMS Delay spread of the radio propagation channel is a measure of the PDP which helps to investigate the system performance. Hence the Cyclic prefix duration is estimated by the expected duration of the multipath channel in the working environment. One way to enlarge spectral efficiency is to adaptively fix the size of the cyclic prefix based on the knowledge of RMS Delay spread in tropical environment during rain events.

The standard LTE cyclic prefix lasts about 5 $\mu$ s and covers a signal path of 1.5 km typically, when the symbol time period is greater than 10 times the RMS delay spread, no ISI equalizer is needed in the LTE receiver [3]. For determining the performance of wireless communication systems in tropical areas, many measurement based channel models and deterministic models were introduced. However these models require complete knowledge of geographical area and if an error occurs, redoing the measurement is equivalent time-consuming. Hence a geometrical based statistical channel models are alternate to model any kind of cellular environment by assuming the scatterer density. The geometry based statistical approach is especially useful for stationary and non-stationary scenarios of BS and MS. This method predicts the channel behaviour for various propagation environments by varying certain predefined channel parameters or scatterer distributions.

The best attractive feature of geometrical channel modeling is to provide the statistical information about the channel, behaviour [4-7]. Many Geometry based scattering model available in the literature helps to build the wireless propagation channel by placing the scatterer in the assumed geometry randomly in 2D or 3D global coordinate system and then finding the AoA(Angle of Arrival) and ToA(Time of Arrival) of multipath. Therefore 2D mathematical based elliptical geometry channel model inducing the rain fading effect is developed to estimate the RMS Delay spread characteristics of the channel for single reflections of signals for various rain fall rates with combined effect of foliage depth and various intensity of rainfall.

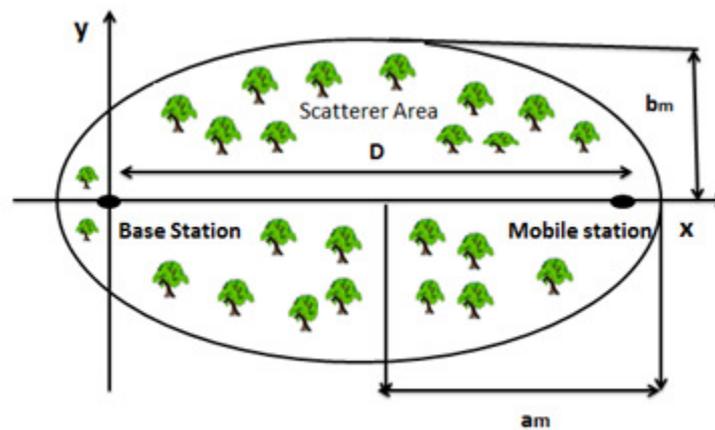
## **2. THE PHYSICAL SCENARIO FOR A SINGLE BOUNCE SCATTERING EFFECT & MODELING ASSUMPTIONS.**

Figure 1 shows the physical scenario of MS and BS deployed in Suburban microcell tropical areas. These areas have dominating trees with broad or tiny leaves. The trees here are considered to be very densely together with the lower tree canopy and the branches and leaves block most of the radio propagation signals from penetrating through it.

In our model the physical scenario is assumed as a College Campus which includes many trees, shrubs and herbs, Green agricultural lands like coorg in Karnataka and many tropical cities located in Karnataka. When there is a rain event in such places, causes attenuation in electromagnetic waves through the process of absorption and scattering limiting the coverage area, and consequently degrading the system performance. Therefore the geometry based statistical channel model incorporating rain into it is developed to investigate the statistical characteristics of the channel.

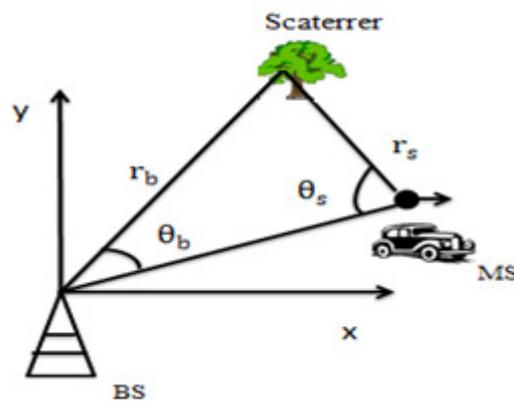


**Figure 1** Propagation Scenario in Sub urban Microcell.



**Figure 2** Propagation Scenario in Sub urban Microcell.

The scatterers i.e trees are confined within an elliptically shaped scattering disc as shown in Fig 2. The maximum foliage depth i.e the distance  $D$  between BS and MS is 1000 mts. The geometry assumed is elliptical scattering disc with its semi-major and semi-minor axes denoted as  $a_m$  and  $b_m$  respectively [8]. The trees which acts as scatterers are assumed to be uniformly distributed in the elliptical scatterer region.



**Figure 3** Tree position in the Global Coordinate System.

The active antenna patterns are omnidirectional for both transmitter and receiver. Received signal at the antenna undergoes no more than one reflection by scatterers when travelling from transmitter to receiver, i.e the model takes into account all the scatterers giving rise to single bounce multipath signal arriving at the receiving antenna before and up to time delay  $\tau_m$ . Hence,  $\tau_m$  is the time difference between first and last multipath signal arrivals at the receiving antenna with signal power greater than some threshold value determined by the system designer. BS is set at the origin of the global coordinate system. The location of any scattering point S is defined by  $(r_b, \theta_b)$  in Cartesian coordinates. The signals received at the antenna are plane waves coming from the horizon hence azimuthal coordinate is considered. In certain environments information about elevation angle may be required such as indoor wireless communication.

The angles  $\theta_b$  and  $\theta_s$  refer to the angles of arrival of the multipath signal at BS and MS respectively as illustrated in the Fig.3. BS is set at the origin of the global coordinate system. In succeeding discussion, the word “x-axis” refers to the x-axis of the global coordinate system. Hence, the location of any scattering point S is defined by  $(x_b, y_b)$ , in Cartesian coordinates as  $(r_b, \theta_b)$  with respect to BS and also MS location is defined by  $(x_s, y_s)$ , in Cartesian coordinates as  $(r_s, \theta_s)$ .

Most of the multipath propagation happens within the canopy layer of the tree, where the rain event effects the propagation path of the direct plane waves more than in any other like trunk and ground layers[9].The model assumes the signal of interest transmitted to be  $\delta(t)$ . It is also assumed that the trees acting as scatterer are uniformly distributed inside the ellipse with BS and MS at the foci. The parameter  $a_m$ , semi major axis of the ellipse and semi minor axis as  $b_m$ , are given by

$$a_m = \frac{c\tau_m}{2\sqrt{\epsilon_r}} \tag{1}$$

$$b_m = \frac{1}{2} \sqrt{\left(\frac{c^2\tau_m^2}{\epsilon_r} - D^2\right)} \tag{2}$$

### 3. MARGINAL TIME OF ARRIVAL (TOA) PDF

The joint AoA/ToA pdf found [10] is invoked in this paper to further investigate the RMS Delay spread for single bounce rain fading effect.

$$f_{\tau,\theta_s}(\tau, \theta_s) = \frac{(\epsilon_r\sqrt{\epsilon_r}D^2c + \sqrt{\epsilon_r}\tau^2c^3 - 2\tau\epsilon_r c^2 D \cos(\theta_s))(\epsilon_r D^2 - \tau^2 c^2)}{4A(\epsilon_r D \cos(\theta_s) - \sqrt{\epsilon_r}\tau c)^3} \tag{3}$$

Integrating the joint AoA/ToA with respect to  $\theta_s$  within the range of  $(0, \pi)$  for the marginal ToA, yields manageable results to obtained marginal ToA for single bounce multipath system. In our statistical model we have considered only the region where the angle of arrival  $0 \leq \theta_s \leq \pi$  because the same holds true for the region where the angle of arrival  $-\pi \leq \theta_s \leq 0$ .The pdf of ToA of the multipath signals from all scattering points within the scattering disc is found by integrating Joint AoA/ToA with respect to  $\theta_s$ , i.e

$$f(\tau) = \int_0^\pi f_{\tau,\theta_s}(\tau, \theta_s) d\theta_s \tag{4}$$

Substituting for  $f_{\tau,\theta_s}(\tau, \theta_s)$  in the above equation yields

$$f(\tau) = \int_0^\pi \frac{(D^2 - \tau^2 c^2)(D^2 c + \tau^2 c^3 - 2\tau c^2 D \cos \theta_s)}{4A(D \cos \theta_s - \tau c)^3} d\theta_s \tag{5}$$

Where  $c = \frac{c}{\sqrt{\epsilon_r}}$  is the delayed LOS (Line of Sight Path) due to rain

$$f(\tau) = (D^2 - \tau^2 c^2) \int_0^\pi \frac{(D^2 c + \tau^2 c^3 - 2\tau c^2 D \cos \theta_s)}{4A(D \cos \theta_s - \tau c)^3} d\theta_s \quad (6)$$

Equation (6) can be expressed in the mathematical form given as [11]

$$\int_0^\pi \frac{w - x \cos \theta_s}{(y \cos \theta_s - z)^3} d\theta_s = \left[ \theta \left( \frac{3xy}{2z^4} \right) - \frac{w}{z^3} - \frac{3wy^2}{z^5} \right]_0^\pi + \left[ \sin \theta \left( \frac{x}{z^3} + \frac{6xy^2}{z^5} - \frac{3wy}{z^4} - \frac{10wy^3}{z^6} \right) \right]_0^\pi + \left[ \sin 2\theta \left( \frac{3xy}{4z^4} - \frac{3wy^2}{2z^5} \right) \right]_0^\pi + \left[ \sin 3\theta \left( \frac{10wy^3}{3z^6} - \frac{2xy^2}{z^5} \right) \right]_0^\pi \quad (7)$$

Substituting  $w = D^2 c + \tau^2 c^3$ ,  $x = 2\tau c^2 D$ ,  $y = D$ ,  $z = \tau c$  and simplifying (7) gives

$$f(\tau) = \left[ (D^2 - \tau^2 c^2) * \pi \left( \frac{3D^2 \tau c^2}{(\tau c)^4} - \frac{D^2 c + \tau^2 c^3}{(\tau c)^3} - \frac{3(D^2 c + \tau^2 c^3) D^2}{(\tau c)^5} \right) \right] \quad (8)$$

Further simplifying (6) by further substitutions and neglecting higher order value, as the maximum angle of arrival  $\theta$  is considered as 180 degrees yields.

$$\int_0^\pi \frac{(D^2 - \tau^2 c^2)(D^2 c + \tau^2 c^3 - 2D\tau c^2 \cos \theta_s)}{(D \cos \theta_s - \tau c)^3} d\theta_s = \pi(D^2 - \tau^2 c^2) \left[ -\frac{D^2}{c^2 \tau^3} - \frac{1}{\tau} - \frac{3D^4}{c^4 \tau^5} \right] = \pi c^2 \left( \frac{D^2}{c^2} - \tau^2 \right) \left[ -\frac{D^2}{c^2 \tau^3} - \frac{1}{\tau} - \frac{3D^4}{c^4 \tau^5} \right] \quad (9)$$

Equating  $\frac{D}{c} = t_{min}$  in (9) yields

$$\int_0^\pi \frac{(D^2 - \tau^2 c^2)(D^2 c + \tau^2 c^3 - 2D\tau c^2 \cos \theta_s)}{(D \cos \theta_s - \tau c)^3} d\theta_s = \pi c^2 (t_{min}^2 - \tau^2) \left[ -\frac{t_{min}^2}{\tau^3} - \frac{1}{\tau} - \frac{3t_{min}^4}{\tau^5} \right] \quad (10)$$

Therefore the Marginal ToApdf of the model observed at MS found from uniform distributed scatterers confined within elliptical scattering disc is further reduces to

$$f(\tau) = \begin{cases} \frac{\pi c^2}{4A} \left[ \frac{2t_{min}^4}{\tau^3} + \tau - \frac{3t_{min}^6}{\tau^5} \right], & \frac{D\sqrt{\epsilon_r}}{c} \leq \tau \leq \sqrt{\epsilon_r} \tau_m \\ 0, & \text{else} \end{cases} \quad (11)$$

Where  $A = \pi a_m b_m$  the area of ellipse, and the propagation speed  $c = \frac{c}{\sqrt{\epsilon_r}}$ .

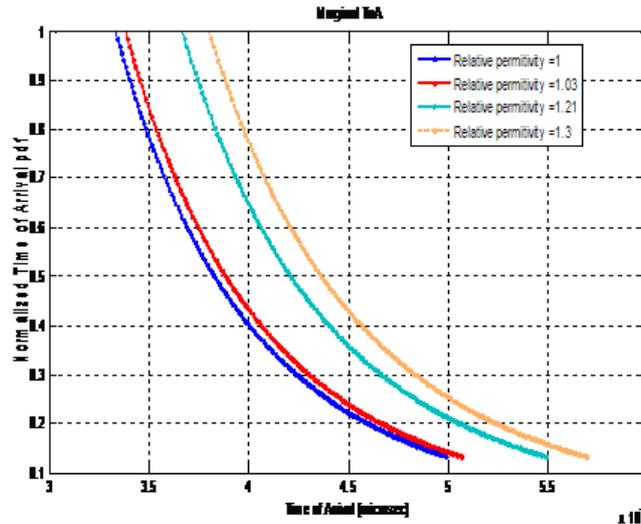


Figure 4 ToApdf of Single Bounce Channel Model

The simulation results shown in the Fig.4 shows the Marginal ToApdf obtained for different rain intensities. Invoking the methods used to describe the different rain rates[10] in our model, it is observed that for no rain event it is assumed the relative permittivity as  $\epsilon_r = 1$ ,  $\epsilon_r = 1.03$  for the rain rate of 0.9mm/h,  $\epsilon_r = 1.21$  for 6.8mm/h and  $\epsilon_r = 1.3$  for 24.4mm/h. Since D is fixed, then the delay of the first multipath signal arrival  $\tau_0$  for  $\epsilon_r = 1$  is also fixed. The value of the semi-major axis of the scattering disc  $a_m = 750$  mts and semi minor axis  $b_m = 550$  mts,  $\tau_{max}$  or  $\tau_m = 5\mu\text{sec}$ , the maximum delay is found to be constant. Figure 4 illustrates the effect of changing the value of relative permittivity  $\epsilon_r$  for various rain events. The curve of the pdf of ToA tends to delay with the corresponding increase in the relative permittivity from 1.03 to 1.3 in case of uniformly distributed scatterers bounded within an elliptical scattering region. It is also observed that the maximum delay increases; this is the case when there will be an increase in rain event influence the multipath signal and hence delays the first multipath signal arrival  $\tau_0$ . Therefore, a change in the value of  $\epsilon_r$  induces changes in first multipath signal arrival  $\tau_0$  as a result, the temporal spread becomes large,  $a_m$  &  $b_m$  remains unchanged. The figure also reveals that increases in dielectric permittivity, increase the maximum delay spread of the multipath component from  $\tau_m$  to  $\sqrt{\epsilon_r}\tau_m$  the tail of the ToApdf. The maximum delay component should be taken into account to estimate the temporal statistics of the multipath signals in cellular environments for various rain events. Maximum Delay component that falls within the receiver sensitivity at the MS must be considered for the Receiver design to detect the signal.

#### 4. RMS DELAY SPREAD FOR SINGLE BOUNCE SCATTERING EFFECT.

For a fixed transmitter and receiver locations, the multipath dispersion is characterized completely by the channel impulse response  $h(t)$ . The RMS delay spread  $\tau_{rms}$  is commonly used to measure the time dispersive nature of multipath. The RMS delay spread is calculated initially using the statistical approach proposed by Janaswamy[12]. According to this approach, the value of RMS delay spread depends upon the Time of arrival of all the multipath components arriving at the receivers. Hence in this work, the RMS delay spread is found which depends on ToApdf along with the relative permittivity of various rain events. The statistics employed here is to first determine the mean and 2nd central moment from the obtained ToApdf to estimate the values of RMS delay spread for various rain events.

According to Rama Krishna Janaswamy, the mean, second order central moment and RMS Delay spread for elliptical scattering model can be found by integrating the marginal ToApdf w.r.t delay component which is given as follows

$$\text{Mean } (\bar{\tau}) = \int_{\tau_{\min}}^{\tau_{\max}} \tau f(\tau) d\tau \quad (12)$$

$$2^{\text{nd}} \text{CM } (\overline{\sigma^2}) = \int_{\tau_{\min}}^{\tau_{\max}} \tau^2 f(\tau) d\tau \quad (13)$$

$$\text{RMS delay spread } (\tau_{rms}) = \sqrt{\overline{\sigma^2} - \bar{\tau}^2} \quad (14)$$

The drastic increase of the number of digital radio communication systems within the forested environment has demanded wide band characterization of outdoor radio channels. Multipath Signal time dispersion is one of the main study issues because it limits the maximum symbol rate that can be used without intersymbol interference [13]. The impulse response (IR) of the radio channel and other parameters, as the mean delay and the root mean square (RMS) delay spread are frequently used to characterize time dispersion nature of the channel within tropical regions. In this work, estimation of RMS Delay spread for tropical region is presented.

The Marginal ToApdf observed at MS of the model using uniform distributed scatterers confined within elliptical scattering disc has been previously derived as:

$$f(\tau) = \begin{cases} \frac{\pi c^2}{4A} \left[ \frac{2t_{\min}^4}{\tau^3} + \tau - \frac{3t_{\min}^6}{\tau^5} \right], \frac{D\sqrt{\epsilon_r}}{c} \leq \tau \leq \sqrt{\epsilon_r} \tau_m & (15) \\ 0, \text{ else} \end{cases}$$

Substituting (15) in (12) and applying Binomial series of expansion the mean can be found as

$$\bar{\tau} = \frac{\pi c^2}{4a_m b_m} \left[ \frac{\tau^3}{3} - 2 \frac{t_{\min}^4}{\tau} + \frac{t_{\min}^6}{\tau^3} \right]_{\tau=\tau_{\min}}^{\tau_{\max}} \quad (16)$$

2nd Central Moment can be computed by substituting (15) in (13) and simplifying mathematically further results as

$$\overline{\sigma^2} = \frac{\pi c^2}{4a_m b_m} \left[ \frac{\tau^4}{4} + 2 \left( t_{\min}^4 \right) \log(\tau) + \frac{3 t_{\min}^6}{2 \tau^2} \right]_{\tau=\tau_{\min}}^{\tau_{\max}} \quad (17)$$

Let  $\tau_{\min} = \frac{D}{c} \sqrt{\epsilon_r}$  and  $\tau_{\max} = \tau_m \sqrt{\epsilon_r}$  yields the RMS delay spread given in (14) for single bounce scatterer rain model . The statistics considered in our channel model for estimating the  $\tau_{rms}$  is much better than the traditional model due to following reasons.

- This approach takes the probability density function of ToA into account by assuming the angle of arrival between 0 and 180 degrees rather finding its cdf(cumulative distribution function) employed in many models available in the literatures.
- The complete scattering area of overlapping with the elliptical geometry is considered, by detremining the arrival time of the LOS path and the final path within the geometry. Thus it can analyze the effects of multipath dispersion effectively and determine the power distribution profile. This approach can also estimate the variations in path loss due to the variation in the rain event. Therefore, the use of entire elliptical geometry for computing the Marginal ToA pdf have shown a tendency of higher delay spreads which in turn results in large RMS delay spread.

In our computations to find RMS Delay spread, the separation between the Tx and the Rx is assumed to be 1000 mts deployed in an elliptical geometry and scatterers i.e trees are assumed to be distributed uniformly within an entire elliptical scattering the region . The RMS delay spread variations with different humidity effects were calculated. Three cases are considered in tropical region. In the first case, for the relative permittivity  $\epsilon_r = 1$  the condition for no rain event, the RMS Delay spread is found to be  $1.13\mu\text{sec}$  where as in Janaswamy model it was found to be  $0.8\mu\text{sec}$ , we can observe a difference of  $0.3\mu\text{sec}$  because our model assumes the complete elliptical scattering region rather than assumng a portion of elliptical scattering region. The second case considers relative permittivity  $\epsilon_r = 1.03$  for the rain event  $R=0.9$  mm/h a low intensity rain,  $\tau_{rms}$  is found to be  $1.15 \mu\text{sec}$  for the moderate rain rate considered to be  $R=6.8\text{mm/h}$  &  $26\text{mm/h}$ , for  $\epsilon_r = 1.21$  &  $1.3$ ,  $\tau_{rms}$  increases respectively. Hence it is required to consider the appropriate value of  $\tau_{rms}$  which helps to fix the length of CPs.

**Table 1** Time Dispersion Parameters for Single bounce scattering model

Relative Permittivity	Mean ( $\mu\text{s}$ )	RMS Delay Spread ( $\mu\text{s}$ )
$\epsilon_r = 1$ (No Rain)	1.5	1.1370
$\epsilon_r = 1.03$	1.522	1.154
$\epsilon_r = 1.21$	1.65	1.25
$\epsilon_r = 1.3$	1.7102	1.2964

Therefore from the table 1 it is clear that the value of RMS delay spread depends on the humidity effect in the tree canopy layer for various rain events. Hence the mathematical model developed investigates that the RMS Delay spread increases for various rain events for a worst case foliage depth of 1000 mts. Conventional OFDM transmission system uses a fixed-length Cyclic Prefix (CP) to counteract Inter-Symbol Interferences (ISI) caused by channel delay spreading under wireless mobile environment. The spectral efficiency and power saving scheme can be employed in OFDM systems by adaptively varying the CP length. Hence based on the statistics derived the length of the CP may be chosen adaptively when the rain rate increases. As a system design rule, the CP length should be about two times the RMS (Root-Mean-Squared) delay spread [14], hence we conclude in this paper that the length of  $\text{CP} = 2 * \tau_{rms}$  for various rain events considered for developing the channel parameters .

## 5. CONCLUSION AND FUTURE WORK

Single bounce scattering effect by trees in tropical area is proposed to find the ToA pdf statistics observed at MS and also found the RMS delay spread of the channel. The proposed method is suitable for the tropical scenerio during various rain intensities. It is achieved by considering the Joint AoA/ToA pdf to find the channel properties and then finding channel delay dispersion parameters. Thus the optimal channel behaviour in case of rain events can be estimated with reduced mathematical complexities. The proposed model with tree as a scatterer element for worst case foliage depth found in many tropical regions can be used to estimate the Cyclic Prefix length, hence increases the spectral efficiency and saves the battery power adaptively depending on the rain intensities. The investigations can be further extended to find the RMS Delay spread in case of double bounce scattering environment especially in dense foliage depth.

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