
ABSTRACT

Removal of noise is an important step in the image restoration process, but denoising of image remains a challenging problem in recent research associate with image processing. Denoising is used to remove the noise from corrupted image, while retaining the edges and other detailed features as much as possible. This noise gets introduced during acquisition, transmission & reception and storage & retrieval processes. We propose an efficient image denoising technique using wavelet based principal component analysis(PCA) with local pixel grouping(LPG).For a better preservation of image local structures, a pixel and its nearest neighbors are modelled as a vector variable, whose training samples are selected from the local window by using block matching based LPG. This method compares PSNR (Peak signal to noise ratio) between original image and noisy image and PSNR between original image and denoised image. The MSE and PSNR of the proposed method and local adaptive wavelet image denoising method are compared and demonstrated. Therefore, the image after denoising has a better visual effect.

KEYWORDS: PCA, Denoising, LPG.

INTRODUCTION

Information transmitted in the form of digital images is becoming a major method of communication in the modern age. An image is often corrupted by noise in its acquisition and transmission. The received image needs processing before it can be used in applications. 'Image Denoising' involves the manipulation of the image data to produce a visually high quality image. Image denoising is used to remove the additive noise while retaining as much as possible the important signal features. In the recent years there has been a fair amount of research on wavelet thresholding and threshold selection for signal de-noising [1], [2],[3], [4], because wavelet provides an appropriate basis for separating noisy signal from the image signal. As a primary low-level image processing procedure, noise removal has been extensively studied and many denoising schemes have been proposed, from the earlier smoothing filters and frequency domain denoising methods [5] to the lately developed wavelet [1–10], curvelet [11] and ridgelet [6] based methods, sparse representation [7] and K-SVD [8] methods, shape-adaptive transform [15], bilateral filtering [10,11], non-local mean based methods [12,13] and non-local collaborative filtering [14].

Wavelet transform (WT) decomposes the input signal into multiple scales, which represent different time-frequency components of the original signal. At each scale thresholding [15,16] and statistical modeling [17-19], can be performed to suppress noise. The processed wavelet coefficients are transformed back into spatial domain by denoising. To represent the image wavelet transform uses a fixed wavelet basis with dilation and translation. Wavelet transform can cause distortion in the denoising output.

To overcome the drawback in wavelet transform, a spatially adaptive principal component analysis (PCA) is used which computes the locally fitted basis to transform the image and a shape-adaptive discrete cosine transform (DCT) to the neighborhood, which can achieve very sparse representation of the image leading to effective denoising. PCA is a useful statistical technique that has found application in fields such as face recognition and image compression,

and is a common technique for finding patterns in data of high dimension.

PCA is a technique that reduces the data into two dimensions. PCA is used abundantly in all forms of analysis - from neuroscience to computer graphics - because it is a simple, non-parametric method of extracting relevant information from confusing data sets. With minimal additional effort PCA provides a roadmap for how to reduce a complex data set to a lower dimension. The other main advantage of PCA is that the data can be compressed by finding the patterns in the data ie. by reducing the number of dimensions, without much loss of information.

In this paper we present an efficient PCA-based denoising method with local pixel grouping (LPG). PCA is a classical de- correlation technique in statistical signal processing used mainly in pattern recognition and dimensionality reduction, etc. [26]. By transforming the original dataset into PCA domain, the noise and trivial information can be removed by preserving only the several most significant principal components. In this paper, a PCA-based scheme is proposed for image denoising by using a moving window to calculate the local statistics, from which the local PCA transformation matrix is estimated. In the denoised output many noise residual and visual distortions appear by applying PCA directly to the noisy image without data selection.

The organization of the paper is as follows: Section 2- System overview that gives a general idea about the overall functioning of the system. Section 3 briefly reviews the procedure of PCA. Section 4 presents the LPG-PCA denoising algorithm in detail. Section 5 presents the experimental results and Section 6 concludes the paper.

SYSTEM OVERVIEW

In this LPG-PCA scheme, we model a pixel and its nearest neighbors as a vector variable. The training samples of this variable are selected by grouping the pixels with similar local spatial structures to the underlying one in the local window. With this LPG procedure, the local statistics of the variables can be accurately computed so that the image edge structures can be well preserved after shrinkage in the PCA domain for noise removal

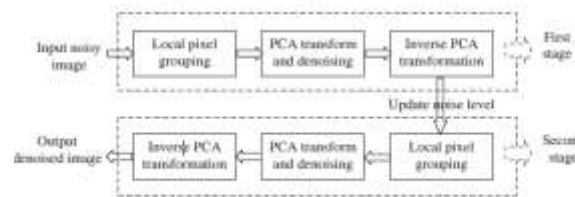


Fig. 1. Flowchart of the proposed two-stage LPG-PCA denoising scheme.

This proposed LPG-PCA algorithm consists of two stages. The first stage yields an initial estimation of the image by removing most of the noise and the second stage will further refine the first stage output. The procedures of both the stages have the same except for the parameter of noise level. Since the noise is significantly reduced in the first stage, the LPG accuracy will be much improved in the second stage so that the final denoising result is visually much better. The proposed LPG-PCA method is a spatially adaptive image representation so that it can better characterize the image local structures.

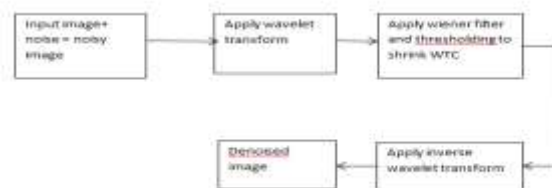


Fig. 2 shows the basic model for denoising of image.

In the implementation of above method, first the noisy image is decomposed by wavelet transform. After this, by using thresholding shrink decomposed images and apply adaptive wiener filter to decomposed images. Finally denoised image is obtained by using inverse wavelet transform.

In the recent year most of the denoising strategies in wavelet domain depend on threshold selection and shrinking of wavelet transform coefficients for image denoising. Wavelet decomposes the image and separate noisy signal from

original signal on appropriate basis in [1]. Adaptive wiener filtering is one of filtering techniques to remove the noise from noisy images. Owing to its simplicity and effectiveness, more attention is considered on wavelet based adaptive wiener denoising in [3].

PRINCIPAL COMPONENT ANALYSIS (PCA)

$Y = [y_1 \ y_2 \ \dots \ y_m]^T$ an m-component vector variable and denoted by

$$Y = \begin{bmatrix} y_{11} & y_{12} & \dots & y_{1n} \\ y_{21} & y_{22} & \dots & y_{2n} \\ y_{m1} & y_{m2} & \dots & y_{mn} \end{bmatrix}$$

the sample matrix of y , where y_{ij} , $j=1,2,\dots,n$, are the discrete samples of variable y_i , $i=1,2,\dots,m$. The goal of PCA is to find an orthonormal transformation matrix P to de-correlate, i.e. $Z = PY$ so that the co-variance matrix of the Z is diagonal. Since the covariance matrix Ω is symmetrical, it can be written as

$$\Omega = \Phi \Lambda \Phi^T$$

where $\Phi = [\Phi_1 \ \Phi_2 \ \dots \ \Phi_m]$ is the m orthonormal eigenvector matrix and $\Lambda = \text{diag}\{\lambda_1, \lambda_2, \dots, \lambda_m\}$ is the diagonal eigenvalue matrix with $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_m$. The terms $\Phi_1, \Phi_2, \dots, \Phi_m$ and $\lambda_1, \lambda_2, \dots, \lambda_m$ are the eigenvectors and eigenvalues of Ω .

By setting $P = \Phi^T$, Y can be decorrelated, i.e. $Z = PY$.

In PCA, the energy of a signal will concentrate on a small subset of the PCA transformed dataset, while the energy of noise will evenly spread over the whole dataset i.e. it fully de-correlates the original dataset, separating signal from noise.

LPG-PCA DENOISING ALGORITHM

4.1. Modeling of spatially adaptive PCA denoising

An image pixel is described by two quantities, the spatial location and its intensity, while the image local structure is represented as a set of neighboring pixels at different intensity levels. The edge structures convey its, edge preservation semantic information of an image which is highly desired in image denoising. In this paper we model a pixel and its nearest neighbors as a vector variable and perform noise reduction on the vector instead of the single pixel. To denoise an underlying pixel, a $K \times K$ window is centered on it and is denoted by $y = [y_1 \ \dots \ y_m]^T$, $m=K^2$, the vector containing all the components within the window. The observed image is corrupted by noise.

We denote it by $yv = y + v$

To estimate y from yv , we view them as (noiseless and noisy) vector variables so that the statistical methods such as PCA can be used. In order to remove the noise from yv by using PCA, we need a set of training samples of yv so that the covariance matrix of y_i and hence the PCA transformation matrix can be calculated.

For this purpose, we use an $L \times L$ ($L > K$) training block centered on yv to find the training samples, as shown in Fig. 2. The simplest way is to take the pixels in each possible $K \times K$ block within the $L \times L$ training block as the samples of noisy variable yv . In this way, there are totally $(L-K+1)^2$ training samples for each component y_{vk} of yv . There can also be very different blocks from the given central $K \times K$ block in the $L \times L$ training window so that taking all the $K \times K$ blocks as the training samples of yv will lead to inaccurate estimation of the covariance matrix of yv , which subsequently leads to inaccurate estimation of the PCA transformation matrix and finally results in much noise residual. Therefore, selecting and grouping the training samples that similar to the central $K \times K$ block is necessary before applying the PCA transform for denoising.

4.2. LPG (Local Pixel Grouping)

For grouping purpose $L \times L$ training window is taken. and a $K \times K$ variable moving window is taken which moves over the training window. There are many techniques for grouping the local pixel like block matching, K-mean clustering ...etc but block matching method is simpler and more suitable.

In yv which is the corrupted image there are possible training blocks within $L \times L$ training window. We denote by y , the column sample vector containing the pixels in the central $K \times K$ block and denote by $y_i, i=1,2,\dots,(L-K+1)^2-1$, the sample vectors corresponding to the other blocks.

4.3 LPG-PCA based denoising

The data set $Yv = Y + V$, where V is the dataset of noise variable v . In denoising process the dataset Yv is centralized by taking the mean value of Ykv . So the mean $\mu_k = \frac{1}{n} \sum_{i=1}^n y_i$. As the noise is having zero mean $Y_k = Y_k - \mu_k$. Now the centralized data set Yv and Y can be obtained. Thus we have $Yv = Y + V$.

The covariance matrix Ω_y of the centralized dataset is calculated. The PCA transformation matrix P_y is obtained.

$$\Omega_{Yv} = \frac{1}{n} (\overline{Y Y^T} + V V^T) = \Omega_y + \Omega_v$$

$$\text{where } \Omega_y = \left(\frac{1}{n}\right) \overline{Y Y^T} \text{ and } \Omega_v = \left(\frac{1}{n}\right) V V^T$$

The component $\Omega(i, j)$ is the correlation between v_i and v_j . Since v_i and v_j are un-correlated for $i \neq j$, we know that Ω_v is a $m \times m$ diagonal matrix with all the diagonal components being σ^2 . In other words, Ω_v can be written as $\sigma^2 I$, where I is the identity matrix. Then it can be readily proved that the PCA transformation matrix P_y associated with Ω_y is the same as the PCA transformation matrix associated with Ω .

We can decompose Ω_y as $\Omega_y = \Phi_y \Lambda_y \Phi_y^T$

where Φ_y is the $m \times m$ orthonormal eigen vector matrix and Λ_y is the diagonal eigenvalue matrix. Since Φ_y is an orthonormal matrix, we can write as $\Omega v = (\sigma^2 I)$

$$= \Phi^{\lambda} (V^{\lambda} + \alpha_s I) \Phi^{\lambda}_T = \Phi^{\lambda} V^{\lambda} \Phi^{\lambda}_T$$

$$\text{where } \Phi^{\lambda} = \Phi^{\lambda} + \Phi^{\lambda} = \Phi^{\lambda} V^{\lambda} \Phi^{\lambda}_T + \Phi^{\lambda} (\alpha_s I) \Phi^{\lambda}_T$$

$$\Phi^{\lambda} (\alpha_s I) \Phi^{\lambda}_T = \Phi^{\lambda} \Phi^{\lambda}_T \Phi^{\lambda}_T$$

which indicates that $\Omega_y v$ and Ω_y have same eigenvector matrix Φ_y .

4.4. Denoising in the second stage

There are mainly two reasons for the noise residual. First, because of the strong noise in the original dataset Yv , the covariance matrix is much noise corrupted, which leads to estimation bias of the PCA transformation matrix and deteriorates the denoising performance. second, the strong noise in the original dataset will also lead to LPG errors, which results in estimation bias of the covariance matrix. Therefore, it is necessary to further process the denoising output for a better noise reduction. Since the noise has been much removed in the first round of LPG-PCA denoising, the LPG accuracy and the estimation of covariance matrix can be much improved with the denoised image. Thus we can implement the LPG-PCA denoising procedure for the second round to enhance the denoising results.

EXPERIMENTAL RESULTS

In the implementation of wavelet denoising we use decomposition technique which divides the image in to different level of sub bands and apply the denoising algorithm.

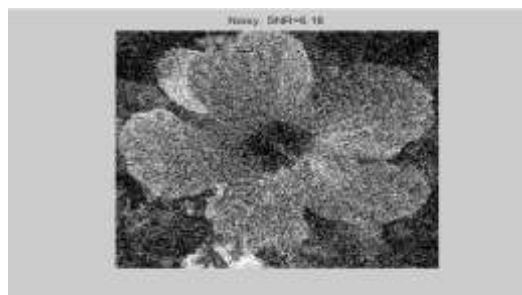


Figure 5.1 Input image +noise data

In the above figure we added the noise data to the input image and we apply the decomposition.

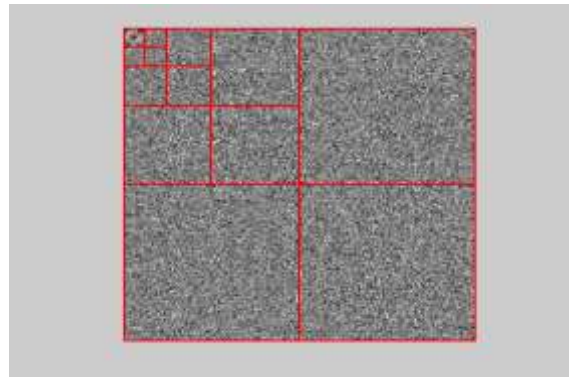


Figure 5.2 decomposition in to subbands



Figure 5.3 Deoised image

In the proposed LPG-PCA denoising algorithm, most of the computational costs depends on LPG grouping and PCA transformation,

Performance comparision of wavelet decomposition and PCA-LPG with different images with variance of 10,20,30 where PSNR and SSIM values are calculated					
		wavelet denoising		PCA-LPG	
		PSNR	SSIM	PSNR	SSIM
lena	s = 10	33.1	0.9154	33.7	0.9243
	s = 20	29.2	0.8455	29.7	0.8605
	s = 30	27.2	0.7878	27.3	0.8066

cameraman	s = 10	33.2	0.917	34.1	0.9356
	s = 20	29.1	0.8449	30.1	0.8902
	s = 30	26.8	0.7945	27.8	0.8558
house	s = 10	34.4	0.8791	35.6	0.9012
	s = 20	31.3	0.8199	32.5	0.8471
	s = 30	29.4	0.7829	30.4	0.8185
paint	s = 10	33	0.927	33.6	0.9311
	s = 20	29	0.8513	29.5	0.8683
	s = 30	26.9	0.7897	27.2	0.8033
Monarch	s = 10	33.1	0.9442	34.2	0.9594
	s = 20	28.8	0.8912	30	0.9202
	s = 30	26.5	0.837	27.4	0.8769

In the implementation of LPG-PCA denoising, actually the complete $K \times K$ block centered on the given pixel will be denoised. Therefore, the finally restored value at a pixel can be set as the average of all the estimates obtained by all windows containing the pixel. We consider a color image and apply the LPC-PCA denoising



Figure 5.4 colour image



Figure 5.5 noisy image



Figure 5.6 denoised using pca-lpg

CONCLUSION

This paper proposed a spatially adaptive image denoising scheme by using principal component analysis (PCA). To preserve the local image structures when denoising, we modeled a pixel and its nearest neighbors as a vector variable, and the denoising of the pixel was converted into the estimation of the variable from its noisy observations. The PCA technique was used for such estimation and the PCA transformation matrix was adaptively trained from the local window of the image. However, in a local window there can have very different structures. The block matching based local pixel grouping (LPG) was used and only the similar sample blocks to the given one are used in the PCA transform matrix estimation. The PCA transformation coefficients were then shrunk to remove noise. The above LPG- PCA denoising procedure was iterated one more time to improve the denoising performance. Our experimental results demonstrated that LPG-PCA can effectively preserve the image fine structures while smoothing noise.

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