

Analysis of whirling speed and Evaluation of self-excited motion of the rotating shaft

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In any rotating element, when the frequency of the rotating element equals the natural frequency of transverse vibration, then that speed is called critical speed of the rotating element. The measurement of this critical speed and related whirling motion is one of the important problems to be addressed by design and maintenance engineers. Also the whirling of shaft and its motion comes under the category of self-excited motion i.e. self-excited vibration in which the exciting forces and inducing motion are controlled by the motion itself.

Keywords: Critical speed, Whirling motion, Self-excited vibration, Amplitude ratio, Frequency ratio.

Introduction

Critical whirling speed: All rotating shafts, even in the absence of external load, will deflect during rotation. The unbalanced mass of the rotating object causes deflection that will create resonant vibration at certain speeds, known as the critical speeds. When a shaft having a rotor or without a rotor, its center of gravity usually doesn't coincide with the axis of rotation of the shaft, this center of gravity is normally displaced from the axis of rotation, although the amount of displacement may be very small. This displacement of C.G. may be due to one or more of the following causes:

1. Eccentricity mounting of the rotor on the shaft.
2. Lack of straightness of the shaft.
3. Bending of shaft under the action of gravity in case of horizontal shaft.
4. Non-homogeneous rotor material.
5. Unbalance magnetic pull in case of electrical machinery.

At the critical speed, the shaft is subjected to violent vibration in the transverse. The excessive vibration, associated with the critical speed, may cause permanent deformation of the shaft of structural damage, for instance blade of rotor of a turbine may come in contact with stator blade. Also the large shaft-deflection at the critical speed, include large bearing reactions and this may lead to bearing failure.

Relation of amplitude ratio w.r.t. frequency ratio:

In beginning, shaft is assumed to be straight. The centrifugal force due to the eccentricity of spinning disc, constitutes the excitation force and shaft is deflected through $s = OS$ as shown in fig.1. This deflection s occurs in the direction of the excitation force with reference to the point O . The restoring force, due to the elastic nature of the shaft is proportional to the deflection and acts in a opposite direction to that of s . The relation between amplitude ratio and frequency ratio is given by;

$$s/e = 1 / [(1/r^2) - 1]$$
Vibration Amplitude with one rotor condition:

When Speed of the rotor is low (means $r < 1$), the amplitude ratio is positive, which implies that dynamic deflection occurs same as eccentricity, so its call **heavy side on outside**. When Speed of the rotor is high (means $r > 1$), the amplitude ratio is negative, which implies that dynamic deflection occurs opposite to as eccentricity, so its call **heavy side on inside**.

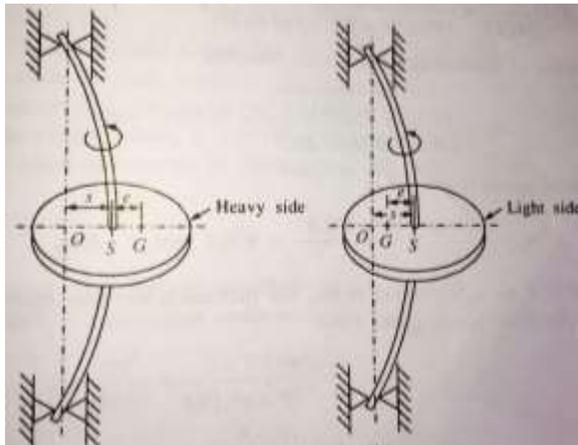


Fig. 1

Objectives

The main objective is to measure the critical speed theoretically first and then compare it with the practical one, and also with this an evaluation of the self-excited motion (vibration) based on the change of amplitude ratio with respect to frequency ratio with the help of experimental set-up. The measurement of critical whirling speed of rotating shaft is done with rotor and without rotor conditions by taking different dimensional approaches.

Procedure

First of all, the shaft is supported and adjusted between the movable bearing housing according to requirement. Then shaft is rotated with the help of motor. Then with the help of the dimmer state, regulate the r.p.m. of motor from initial speed to critical speed and so on. When the initial vibration occurs, the deflection starts in the dial indicator, and at that same time; measure the speed of the rotating shaft with the help of contact-type tachometer. And this speed is later compared with the theoretical data.

Along with this, an evaluation of self-excited motion or vibration is also done by gradually increasing the rotational speed of the shaft with the help of dimmer; and finally when frequency ratio $r \gg 1$ i.e. shaft speed is very much higher than the natural frequency of transverse vibration, at that time amplitude ratio approaches 1 from negative side.

Result analysis

In experimental measurement of critical speed of the rotating shaft, 10mm and 16mm diameter shafts are used for two different conditions i.e. with rotor and without rotor.

Diameter of the shaft is 10 mm and with rotor:

Diameter of the shaft $d = 10$ mm
 Mass of the rotor and shaft $m = 0.600$ kg
 Length of the shaft $l = 0.950$ m
 Eccentricity $e = 0.3 \times 10^{-3}$ m
 Modulus of elasticity $E = 2.05 \times 10^{11}$ N/m²

Theoretical Critical Speed analysis:-

- **Static deflection of simply supported shaft:-**

$$\delta = \frac{wl^3}{48EI}$$

$$= \frac{m \cdot g \cdot l^3}{48 \cdot E \cdot d^4 \cdot \frac{\pi}{64}}$$

$$= \frac{0.600 \cdot 9.81 \cdot 64 \cdot 0.950^3}{48 \cdot 2.06 \cdot 3.14 \cdot 10^{11} \cdot 0.01^4}$$

$\delta = 1.04 \times 10^{-3}$ m

- **Theoretical critical speed:-**

$$(N)_{th} = \frac{30}{\pi} \sqrt{\frac{g}{\delta}}$$

$(N)_{th} = 928$ r.p.m

- **Critical speed measured with the help of dial gauge and tachometer on experiment set up:-**

$(N)_{practical} = 990$ r.p.m

Diameter of the shaft is 10 mm and without rotor condition:

Diameter of the shaft $d = 10$ mm
 Mass of the shaft $m = 0.485$ kg
 Length of the shaft $l = 0.950$ m
 Eccentricity $e = 0.2$ mm = 0.2×10^{-3} m
 Modulus of elasticity $E = 2.05 \times 10^{11}$ N/m²

Theoretical Critical Speed analysis :-

- **Static deflection of simply supported shaft:-**

$$\delta = \frac{wl^3}{48EI}$$

$$= \frac{m \cdot g \cdot l^3}{48 \cdot E \cdot d^4 \cdot \frac{\pi}{64}}$$

$$= \frac{0.485 \times 9.81 \times 64 \times 0.950^3}{48 \times 2.06 \times 3.14 \times 10^{11} \times 0.01^4}$$

$$\delta = 8.41 \times 10^{-3} \text{ m}$$

➤ **Theoretical critical speed:-**

$$(N)_{th} = \frac{30}{\pi} \sqrt{\frac{g}{\delta}}$$

$$(N)_{th} = 1031 \text{ r.p.m.}$$

➤ **Critical speed measured with the help of Tachometer and dial gauge on experimental set-up:-**

$$(N_c)_{practical} = 1010 \text{ r.p.m.}$$

Diameter of the shaft is 16mm and with rotor condition:-

Mass of the shaft $m = 1.652 \text{ kg}$

Length of the shaft $l = 0.900 \text{ m}$

Eccentricity $e = 0.25 \text{ mm} = 0.25 \times 10^{-3} \text{ m}$

➤ **Static deflection of simply supported shaft:-**

$$\delta = \frac{wl^3}{48EI}$$

$$\delta = 3.72 \times 10^{-4} \text{ m}$$

➤ **Theoretical critical speed :-**

$$(N_c)_{th} = \frac{30}{\pi} \sqrt{\frac{g}{\delta}}$$

$$(N_c)_{th} = 1552.35 \text{ r.p.m.}$$

➤ **Speed measured with the help of dial gauge and tachometer:-**

$$(N_c)_{practical} = 1610 \text{ r.p.m.}$$

Diameter of shaft is 16mm and without rotor condition:-

Mass of the rotor and shaft $m = 1.190 \text{ kg}$

➤ **Static deflection of simply supported shaft**

$$\delta = \frac{Wl^3}{48EI}$$

$$= \frac{m \cdot g \cdot l^3}{48 \cdot E \cdot I}$$

$$= \frac{1.190 \times 9.8 \times 64 \times (0.900)^3}{48 \times 2.06 \times 3.14 \times (0.016)^4 \times 10^{11}}$$

$$\delta = 2.676 \times 10^{-4} \text{ m}$$

➤ **Theoretical critical speed :-**

$$(N_c)_{th} = \frac{30}{\pi} \sqrt{\frac{g}{\delta}}$$

$$(N_c)_{th} = 1829 \text{ r.p.m.}$$

➤ **Critical speed measured with the help of dial gauge and tachometer:**

$$(N_c)_{prc} = 1910 \text{ r.p.m.}$$

As we gradually increase the speed of the rotational shaft and set up the position when frequency of rotation is very much higher than the natural frequency of transverse vibration, physically this implies that the phase angle is 180 degrees and the dynamic deflections of the shaft equals the eccentricity but is on the negative side of the bearing centre line. In other words, the centre of gravity **G** lies on the bearing centre line and the shaft becomes stable and runs very smoothly. The graphical representation as shown in fig. 2 gives the relation between Amplitude ratio and rotational speed of the shaft in rpm.

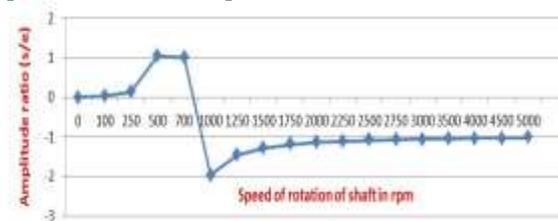


Fig.2

Conclusion

The measurement of the critical speed experimentally gives a nearer reading, when compare it with the theoretical one. Also an evaluation of the self-excited motion (vibration) based on the change of amplitude ratio with respect to frequency ratio with the help of experimental set-up is done successfully.

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