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L-SASAKIAN MANIFOLDS WITH SEMI-SYMMETRIC CONNECTION

L K Pandey
D S Institute of Technology & Management, Ghaziabad (U.P.) 201007, India

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ABSTRACT

The purpose of this paper is to study Lorentzian Sasakian manifolds [1] and generalized Lorentzian Co-symplectic manifolds [2] with semi-symmetric connection [3] [4].

KEYWORDS: Lorentzian Sasakian manifolds, generalized Lorentzian-Co-symplectic manifolds, semi-symmetric connection.

INTRODUCTION

An n-dimensional differentiable manifold M_n, on which there are defined a tensor field $F$of type (1, 1), a vector field T, a 1-form A and a Lorentzian metric g, satisfying for arbitrary vector fields $X, Y, Z, \ldots$

(1.1) $\overline{\overline{X}} = -X - A(X)T, \overline{T} = 0, A(T) = -1, \overline{X} \stackrel{\text{def}}{=} FX, A(\overline{X}) = 0$, rank $F = n-1$

(1.2) $g(\overline{X}, \overline{Y}) = g(X, Y) + A(X)A(Y)$, where $A(X) = g(X, T)$,

$`F(X, Y) \stackrel{\text{def}}{=} g(\overline{X}, Y) = -`F(Y, X)$,

(1.3) (a) $(D_X`F)(Y, Z) + A(Y)\, g(\overline{Z}, \overline{X}) - A(Z) g(\overline{X}, \overline{Y}) = 0$ (b) $D_X T = \overline{X}$

Then M_n is called a Lorentzian Sasakian manifold (an L-Sasakian manifold).

In an L-Sasakian manifold, we have

(1.4) (a) $(D_X A)(Y) = `F(X, Y)$ $\Leftrightarrow$ (b) $(D_X A)(\overline{Y}) = g(\overline{X}, \overline{Y})$

Nijenhuis tensor in an L-Contact manifold is given by

(1.5) $`N(X, Y, Z) = (D_{\overline{X}}`F)(Y, Z) + (D_{\overline{Y}}`F)(Z, X) + (D_X`F)(Y, \overline{Z}) + (D_Y`F)(\overline{Z}, X)$

Where

$$`N(X, Y, Z) \stackrel{\text{def}}{=} g(N(X, Y), Z)$$

NEARLY AND ALMOST LORENTZIAN SASAKIAN MANIFOLDS

An L-contact manifold is called a nearly Lorentzian Sasakian manifold (a nearly L-Sasakian manifold), if

(2.1) $(D_X`F)(Y, Z) + A(Y)\, g(\overline{Z}, \overline{X}) - A(Z) g(\overline{X}, \overline{Y})$

$= (D_Y`F)(Z, X) + A(Z)\, g(\overline{X}, \overline{Y}) - A(X) g(\overline{Y}, \overline{Z})$

$= (D_Z`F)(X, Y) + A(X)\, g(\overline{Y}, \overline{Z}) - A(Y) g(\overline{Z}, \overline{X})$

An L-Contact manifold is called an almost Lorentzian Sasakian manifold (an almost L-Sasakian manifold), if

(2.2) $(D_X`F)(Y, Z) + (D_Y`F)(Z, X) + (D_Z`F)(X, Y) = 0$

SEMI-SYMMETRIC CONNECTION

Let us consider a connection B on M_n, defined by

(3.1) $B_X Y \stackrel{\text{def}}{=} D_X Y - A(X)Y$

The torsion tensor S of B is given by

(3.2) $S(X, Y) = A(Y)X - A(X)Y$

Also, $(B_X g)(Y, Z) = 2A(X) g(Y, Z)$, then B is called a semi-symmetric connection.

Let

(3.3) (a) $B_X Y = D_X Y + H(X,Y)$

Where H is a tensor field of type (1, 2), then

(b) $H(X,Y) = -A(X)Y$

(c) $`H(X,Y,Z) = -A(X)g(Y,Z)$

(d) $`S(X,Y,Z) = `H(X,Y,Z) - `H(Y,X,Z)$

Where

$$`H(X,Y,Z) \stackrel{\text{def}}{=} g(H(X,Y),Z)$$
$$`S(X,Y,Z) \stackrel{\text{def}}{=} g(S(X,Y),Z)$$

Hence

An L-contact manifold will be an L-Sasakian manifold, if

(3.4) $(B_X `F)(Y,Z) - 2A(X)`F(Y,Z) + A(Y)\, g(\overline{Z},\overline{X}) - A(Z)g(\overline{X},\overline{Y}) = 0$

An L-contact manifold will be a nearly L-Sasakian manifold, if

(3.5) (a) $(B_X `F)(Y,Z) - 2A(X)`F(Y,Z) + A(Y)\, g(\overline{Z},\overline{X}) - A(Z)g(\overline{X},\overline{Y})$

$= (B_Y `F)(Z,X) - 2A(Y)`F(Z,X) + A(Z)\, g(\overline{X},\overline{Y}) - A(X)g(\overline{Y},\overline{Z})$

$= (B_Z `F)(X,Y) - 2A(Z)`F(X,Y) + A(X)\, g(\overline{Y},\overline{Z}) - A(Y)g(\overline{Z},\overline{X})$

(b) $(B_{\overline{X}} `F)(\overline{Y}, \overline{Z}) = (B_{\overline{Y}} `F)(\overline{Z}, \overline{X}) = (B_{\overline{Z}} `F)(\overline{X}, \overline{Y})$.

This implies

(3.6) $(B_X `F)(Y,Z) - (B_Y `F)(Z,X) + A(X)g(\overline{Y},\overline{Z}) - 2A(X)`F(Y,Z) + A(Y)\, g(\overline{Z},\overline{X}) + 2A(Y)`F(Z,X) - 2A(Z)g(\overline{X},\overline{Y}) = 0$

These equations can be modified as

(3.7) $(B_X `F)(\overline{Y},Z) - (B_{\overline{Y}} `F)(Z,X) + 2A(X)g(\overline{Y},\overline{Z}) + A(X)`F(Y,Z) + 2A(Z)`F(X,Y) = 0$

(3.8) $(B_X `F)\left(\overline{\overline{Y}},Z\right) - \left(B_{\overline{\overline{Y}}} F\right)(Z,X) + 2A(X)`F(Y,Z) - A(X)g(\overline{Y},\overline{Z}) + 2A(Z)g(\overline{X},\overline{Y}) = 0$

An L-Contact manifold is called an almost L-Sasakian manifold, if

(3.9) $(B_X `F)(Y,Z) + (B_Y `F)(Z,X) + (B_Z `F)(X,Y) - 2\{A(X)`F(Y,Z) + A(Y)`F(Z,X) + A(Z)`F(X,Y)\} = 0$

This gives

(3.10) $(B_{\overline{X}} `F)(\overline{Y}, \overline{Z}) + (B_{\overline{Y}} `F)(\overline{Z}, \overline{X}) + (B_{\overline{Z}} `F)(\overline{X}, \overline{Y}) = 0$

An L-Contact manifold is called a generalized L-Co-symplectic manifold, if

(3.11) $(B_X `F)(Y,Z) + A(Y)(B_X A)(\overline{Z}) - A(Z)(B_X A)(\overline{Y}) - 2A(X)`F(Y,Z) = 0$

An L-Contact manifold is called a generalized nearly L-Co-symplectic manifold, if

(3.12) $(B_X `F)(Y,Z) + A(Y)(B_X A)(\overline{Z}) - A(Z)(B_X A)(\overline{Y}) - 2A(X)`F(Y,Z)$

$= (B_Y `F)(Z,X) + A(Z)(B_Y A)(\overline{X}) - A(X)(B_Y A)(\overline{Z}) - 2A(Y)`F(Z,X)$

$= (B_Z `F)(X,Y) + A(X)(B_Z A)(\overline{Y}) - A(Y)(B_Z A)(\overline{X}) - 2A(Z)`F(X,Y)$

An L-Contact manifold is called a generalized almost L-Co-symplectic manifold, if

(3.13) (a) $(B_X `F)(Y,Z) + (B_Y `F)(Z,X) + (B_Z `F)(X,Y) - A(X)\{(B_Y A)(\overline{Z}) - (B_Z A)(\overline{Y})\}$

$-A(Y)\{(B_Z A)(\overline{X}) - (B_X A)(\overline{Z})\} - A(Z)\{(B_X A)(\overline{Y}) - (B_Y A)(\overline{X})\} -$

$2\{A(X)`F(Y,Z) + A(Y)`F(Z,X) + A(Z)`F(X,Y)\} = 0.$

This implies

(b) $(B_{\overline{X}} `F)(\overline{Y}, \overline{Z}) + (B_{\overline{Y}} `F)(\overline{Z}, \overline{X}) + (B_{\overline{Z}} `F)(\overline{X}, \overline{Y}) = 0$

Theorem 3.1 A generalized L-Co-symplectic manifold is an L-Sasakian manifold, if

(3.14) $B_X T = X + \overline{X} + \overline{\overline{X}}$

Proof. The result directly follows from (3.4) and (3.11).

Theorem 3.2 A nearly L-Sasakian manifold is a generalized nearly L-Co-symplectic manifold, in which

(3.15) (a) $(B_X A)(\overline{Y}) = g(\overline{X},\overline{Y}) \Leftrightarrow$ (b) $(B_X A)(Y) = -g\left(X, Y + \overline{Y} + \overline{\overline{Y}}\right) \Leftrightarrow$

(c) $B_X T = X + \overline{X} + \overline{\overline{X}}$

Proof. Using (3.5) (a) and (3.12), the result follows by simple computation.

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Theorem 3.3 A generalized almost L-Co-symplectic manifold is an almost L-Sasakian manifold, if

(3.16) $(B_X A)(\overline{Y}) = (B_Y A)(\overline{X}) = g(\overline{X}, \overline{Y})$

Proof. Making the use of (3.9) and (3.13) (a), we get (3.16).

In an L-Contact manifold with the semi-symmetric connection B, Nijenhuis tensor is given by

(3.17) $`N(X,Y,Z) = (B_{\overline{X}}`F)(Y,Z) + (B_{\overline{Y}}`F)(Z,X) + (B_X`F)(Y,\overline{Z}) + (B_Y`F)(\overline{Z},X) - 2A(X)g(Y,Z) + 2A(Y)g(Z,X)$

CURVATURE TENSORS

Let R be the curvature tensor of the connection B and K be the curvature tensor of the connection D, then in an L-Contact manifold, we have

(4.1) (a) $R(X,Y,Z) = K(X,Y,Z) - (D_X A)(Y)Z + (D_Y A)(X)Z \Leftrightarrow$

(b) $R(X,Y,Z) = K(X,Y,Z) - (B_X A)(Y)Z + (B_Y A)(X)Z$

In an L-Sasakian manifold, we have

(4.2) (a) $R(X,Y,Z) = K(X,Y,Z) - 2\,`F(X,Y)Z.$

(b) $K(X,Y,T) = -A(Y)X + A(X)Y = -S(X,Y) \Leftrightarrow$

(c) $`K(X,Y,T,Z) = -A(Y)g(Z,X) + A(X)g(Y,Z) \Leftrightarrow$

(d) $`K(X,Y,Z,T) = -A(X)g(Y,Z) + A(Y)g(Z,X) \Leftrightarrow$

(e) $K(X,Y,Z) = -g(Y,Z)X + g(Z,X)Y \Leftrightarrow$

(f) $`K(X,Y,Z,W) = -g(Y,Z)g(W,X) + g(Z,X)g(Y,W)$

After some computation, we get

(4.3) (a) $K(X,Y,\overline{Z}) - \overline{K(X,Y,Z)} = -`F(Y,Z)\overline{\overline{X}} - `F(Z,X)\overline{\overline{Y}} + g(\overline{Y}, \overline{Z})\overline{X} - g(\overline{Z}, \overline{X})\overline{Y} - A(X)`F(Y,Z)T - A(Y)`F(Z,X)T - A(Y)A(Z)\overline{X} + A(Z)A(X)\overline{Y} \Leftrightarrow$

(b) $`K(X,Y,\overline{Z},W) - `K(X,Y,Z,\overline{W}) = `F(Y,Z)g(\overline{W}, \overline{X}) + `F(Z,X)g(\overline{Y}, \overline{W}) + `F(Y,W)g(\overline{Z}, \overline{X}) + `F(W,X)g(\overline{Y}, \overline{Z}) - A(W)A(X)F(Y,Z) - A(Y)A(W)`F(Z,X) - A(Y)A(Z)`F(W,X) - A(Z)A(X)`F(Y,W).$

Also,

(4.4) $`K(X,Y,\overline{Z},\overline{W}) - `K(X,Y,Z,W) = `F(Y,Z)`F(W,X) - `F(Z,X)`F(Y,W) + g(\overline{Y}, \overline{Z})g(\overline{W}, \overline{X}) - g(\overline{Z}, \overline{X})g(\overline{Y}, \overline{W}) - A(W)A(X)g(\overline{Y}, \overline{Z}) - A(Y)A(Z)g(\overline{W}, \overline{X}) + A(Z)A(X)g(\overline{Y}, \overline{W}) + A(Y)A(W)g(\overline{Z}, \overline{X}).$

And

(4.5) $`K(\overline{X},\overline{Y},\overline{Z},\overline{W}) - `K(X,Y,\overline{Z},\overline{W}) = -g(\overline{Y}, \overline{Z})g(\overline{W}, \overline{X}) + g(\overline{Z}, \overline{X})g(\overline{Y}, \overline{W}) - F(Y,Z)`F(W,X) + `F(Z,X)`F(Y,W)$

Adding (4.4) and (4.5), we get

(4.6) $`K(\overline{X},\overline{Y},\overline{Z},\overline{W}) - `K(X,Y,Z,W) = -A(W)A(X)g(Y,Z) - A(Y)A(Z)g(W,X) + A(Z)A(X)g(Y,W) + A(Y)A(W)g(Z,X)$

This equation is equivalent to

(4.7) $\overline{K(\overline{X},\overline{Y},\overline{Z})} + K(X,Y,Z) - A(X)g(Y,Z)T - A(Y)A(Z)X + A(Y)g(Z,X)T + A(Z)A(X)Y = 0.$

COMPLETELY INTEGRABLE MANIFOLDS

Barring X, Y, Z in (3.17) and using equations (3.5) (b), we see that a nearly L-Sasakian manifold is completely integrable, if

(5.1) $(B_{\overline{\overline{X}}}`F)(\overline{Y}, \overline{Z}) = (B_{\overline{\overline{Y}}}`F)(\overline{X}, \overline{Z}).$

Barring X, Y, Z in (3.17) and using equations (3.10), we see that an almost L-Sasakian manifold is completely integrable, if

(5.2) $(B_{\overline{Z}}`F)(\overline{X}, \overline{\overline{Y}}) = 0.$

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