

# POLE ASSIGNMENT FOR GLASS CAPILLARY TUBE DRAWING PROCESS BY USING MATLAB AND MAPLE LANGUAGE

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## Abstract:

The question of pole placement for glass capillary tube drawing process is considered. It is used a frequency domain approach to an arbitrary finite spectrum assignment for multivariable time delay systems in order to control glass capillary tube drawing process. The Padé approximation is used for the system of third order and time delay is eliminated from the transfer function of the process. The responses are shown as well the transfer function of the closed loop after applying finite spectrum method. By choosing state variables it is obtained non degenerative transfer function of process model. The time delays in open loop remained the same as in the closed loop. The all poles are located in the left half plane and system is stable. Appropriate program support for this type of problems is developed in Maple language.

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Glass capillary, pole placement, time delay, stability, finite spectrum, mathematical model, frequency domain, Pade approximation.

## 1. INTRODUCTION

Howver despite the fact that the drawing of glass capillary tubes is well known technological process, and that the process of drawing of optical fibers is good described in the literature, there is yet not many references relating to this specific field, and as well detailed analyze and synthesis in the frequency domain of this phenomena [3-5]. In brief, the glass capillary tube is produced by heating an end of the hollow cylindrical glass preform and then by drawing out melted part thereof which contracts first quickly and then slowly during cooling process [6-7].

The glass capillary tube takes part at the relatively low temperatures in contrast with drawing process of optical fibre [11]. At low temperatures the viscous stresses are dominant, and thickness of a wall of the capillary tybe should be controlled [12].

The studies of Geyling (1976), Clermont (1984), Mayers (1989) and Papamichael and Miaulis (1990) were extremely important and by them was given the significant contribution to explaining and describing whole process [15]. Taking into account that a very rapid change of the viscosity is very

complex problem, the best way is to describe the drawing process qualitatively.

There are various processes of drawing glass tubes from a source of molten glass such as Vello, Danner and Down, and the shape of the glass tube is characterized by the diameter of cross sectional area of the tube and the thickness of the wall.

The geometry of the tube may be circular, rectangular or square, and there are different geometry techniques for obtaining required results.

Sometimes it is used inversion problem such as in manufacturing of non-axisymmetric capillary tubing where the necessity for determination the diametar of the shape is required to achieve the final shape.

Mathematical models of the process are usually complex and described by nonlinear partial different equations of higher order [1].

In order to present valid mathematical method time delay which is presented in the system should not be neglected [2]. In time delay systems, delay might occur in state, control and as well in state and in control, when arises the principal difficulty in the control loop, such as increased phase lag

which leads to unstable control system behaviour at relatively low controller gains [2]. These problems are getting even more complex in case of distributed parameter systems with time delay or hereditary systems such as chemical reactors or paper making machine [13].

The aim of this paper is:

- a) Application of finite spectrum assignment method on glass capillary tube model, and
- b) Pole placement on arbitrary chosen locations, Padé approximation and system elimination of time delay, and stability checking.

## 2. EXPERIMENT APPARATUS AND PROCESS DESCRIPTION

### 2.1. Experiment apparatus

Taking into account the procedure for forming glass tube we can distinguish procedure of constantly pulling the glass material from a vessel of a molten glass, and conventional "tank drawn" glass tubing [15].

System for measuring and drawing glass capillary consists of:

1. motive mechanism;
2. control of reducing speed of perform;
3. furnace;
4. winding plant;
5. control of drawing capillary speed;
6. signal analyser;
7. winding drum.

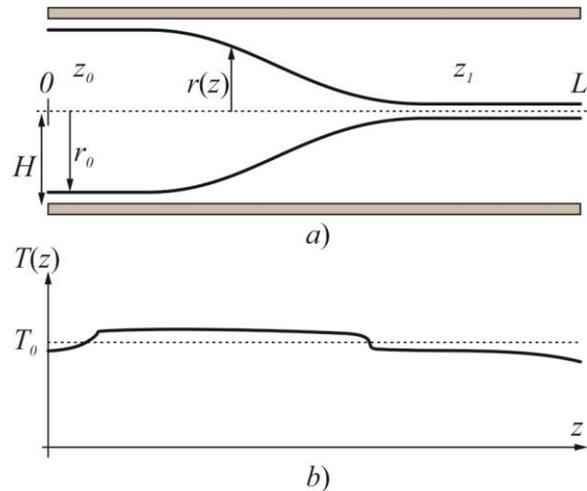
In the figure 1, it is presented the initial glass pre-form with initial radius  $a_0$  inside the furnace with the radius  $b$ . At the top of the furnace is located the origin, and  $z$  axis has positive direction from above to the bottom of the furnace, and the length of the furnace is denoted with  $L$ .

In order to simplify process three basic zones of the process are discussed separately. The temperature of the pre-form in the first zone is lower than the softening temperature of glass, and appropriate part of the pre-form is considered to be a solid body.

The drawn down zone where the temperature of the pre-form exceeds the softening temperature of the glass is the second zone. The pre-form is considered to be a very viscous fluid and it contracts almost to its final value. The appropriate part of the pre-form is also considered to be a very viscous fluid, and it is caused by draw down zone of the capillary tube, where the temperature of the pre-form corresponds to the temperature of the pre-form in the forming zone.

In the cooling zone or the third zone, the temperature of the drawn capillary tube descends

under the freezing temperature of glass causing the capillary tube to act as solid body. The temperature and mass changes were registered on personal computer. Here it could be seen that this zone extends through the lower part of the furnace, and ends at the measuring device for measuring of the diameter of the drawn capillary tube placed above the pair of rollers pulling out the capillary tube.



a) Cross sectional view of pre-form drawing system;  
b) Temperature profile of furnace.

Fig. 1. Initial glass pre-form with initial radius

### 2.2. Process description

Mathematical model of the process is depicted in details in [15], and it is based on assumptions and balance equations and presented with non-linear partial differential equations.

Main characteristic of the process in all three zones is time lag. In first steps it is very important to evaluate the axial coordinates of the softening point and as well the freezing point of the glass, and velocity profile of the second zone. The second zone is described with partial differential equations which were solved numerically. According to the experiment given in [15], coordinate of the freezing point could be obtained by solving differential equations. For getting radius, temperature and velocity it is used a standard shooting method and boundary conditions in steady state.

In system is present information delay and process has distributed component, so it is essential to choose control locations. The onset point of the drawing force is the first adopted location in the right boundary of the second zone, and second location is in the drawing zone controlled by the heater.

### 3. PROCESS MODEL IN FREQUENCY DOMAIN

Considerations about the process were furnished by Sarboh [15] and Milinkovic [11], and they showed considerable time delay present in the system. The earlier methods of the system included time-delay compensators such as Smith predictors for scalar systems, were unable to stabilize open loop unstable systems. The new methods for stabilization of such systems were introduced such as [8-9], where it has been shown that by converting the infinite spectrum problem into finite one through the removal of all delays from the characteristic equation, give improved efficiency and accuracy [10].

The question of choosing control locations and pole assignment has been considered, taking into account that it is distributed parameter process with information delay. The right boundary of the second zone, the onset point of the drawing zone is first adopted location, and the other location is distributed inside the drawing zone [7].

The transfer function of one point that belongs to drawing zone is given with [15]:

$$G = \begin{bmatrix} \frac{-8.4 \cdot 10^{-10}}{s+0.211} \exp(-20.31s) & \frac{-1.32 \cdot 10^{-10}}{s+1442.5} \exp(-30s) \\ \frac{-0.88 \cdot 10^{-10}}{s+0.211} \exp(-20.31s) & \frac{-3.16 \cdot 10^{-10}}{s+1442.5} \exp(-30s) \end{bmatrix} \quad (1)$$

It is obtained the transfer function with no delays:

$$G^0 = \begin{bmatrix} \frac{-20.62 \cdot 10^{-10}}{s+0.211} & \frac{-1.32 \cdot 10^{-10}}{s+1442.5} \\ \frac{-3.2 \cdot 10^{-10}}{s+0.211} & \frac{-4.2 \cdot 10^{-10}}{s+1442.5} \end{bmatrix} \quad (2)$$

$$G^0 = \begin{bmatrix} -20.62 \cdot 10^{-10} & -0.684 \cdot 10^{-10} \\ -3.2 \cdot 10^{-10} & -4.2 \cdot 10^{-10} \end{bmatrix} \cdot \begin{bmatrix} s+0.211 & 0 \\ 0 & s+1442.5 \end{bmatrix}^{-1} \quad (3)$$

The polynomial matrix is presented with:

$$P_c = \begin{bmatrix} s+2 & 0 \\ 0 & s+1442.5 \end{bmatrix}, Q = \begin{bmatrix} s+10 & 0 \\ 0 & s+1442.5 \end{bmatrix}^{-1} \quad (4)$$

The non-degenerative transfer function of the process model is obtained by using the method of choosing the state variables [14-15], and it could be remarked that time delays in closed-loop transfer matrix remain the same as in open loop, and the numerators remained the same too.

The closed-loop transfer matrix is given in the form shown down below:

$$G_{yv} = \begin{bmatrix} \frac{-8.4 \cdot 10^{-10}}{s+3} \exp(-20.31s) & \frac{-1.32 \cdot 10^{-10}}{s+1442.5} \exp(-30s) \\ \frac{-0.88 \cdot 10^{-10}}{s+2} \exp(-20.31s) & \frac{-3.16 \cdot 10^{-10}}{s+1442.5} \exp(-30s) \end{bmatrix} \quad (5)$$

Model of the process with time delay is by using Pade approximation presented with the following equation:

$$\exp(-\tau s) = \frac{2 - \tau \cdot s + \frac{(-\tau s)^2}{2!} + \frac{(-\tau s)^3}{3!}}{2 + \tau \cdot s + \frac{(-\tau s)^2}{2!} + \frac{(-\tau s)^3}{3!}} \quad (6)$$

### 4. RESULTS AND DISCUSIONS

There are few well known phases in eliminating the time delay from the system in state space [12], such as few options for Padé approximations in Matlab language in state space and in frequency domain. These options are presented with the following equations [13]:

$$[num, den] = pade(\tau, n) \quad (7)$$

$$[A, B, C, D] = pade(\tau, n) \quad (8)$$

where  $\tau$  represents time delay, and  $n$  is degree of the polynomial which is used for applied approximation.

First option as result gives system model in transfer function form, and the other in state space.

The graphical representations of the responses are shown in the figure 2.

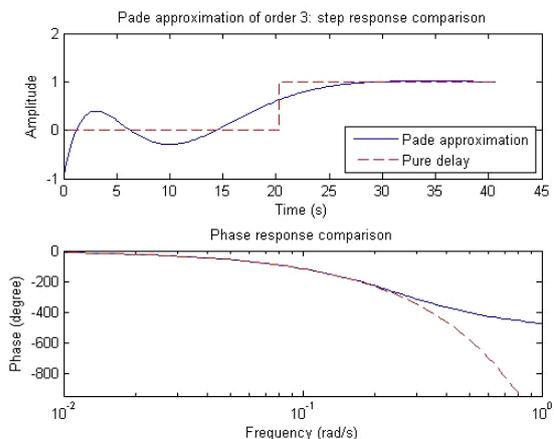


Fig. 2. Graphical representation of step response for first time delay approximation

The graphical representation for the other time delay is presented in Figure 3.

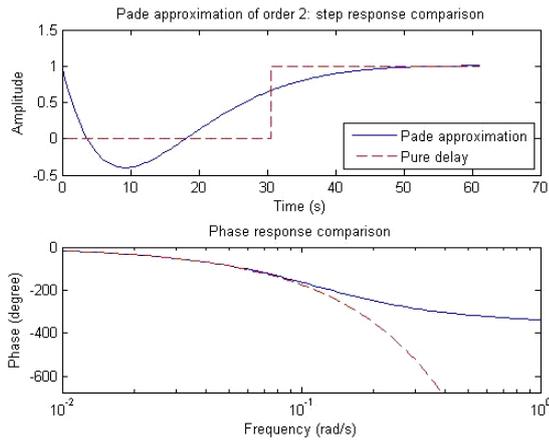


Fig. 3. Graphical representation of step response for second time delay approximation

$$G_{yv} = \begin{bmatrix} \frac{-8.4 \cdot 10^{-10}}{s+3} & \frac{-1.32 \cdot 10^{-10}}{s+1442.5} \\ \frac{-0.88 \cdot 10^{-10}}{s+2} & \frac{-3.16 \cdot 10^{-10}}{s+1442.5} \end{bmatrix} \begin{bmatrix} \left( \frac{-s^3 + 0.59s^2 - 0.14s + 0.014}{s^3 + 0.59s^2 + 0.14s + 0.01} \right) & \left( \frac{-s^3 + 0.819s^2 - 0.34s + 0.0119}{s^3 + 0.81s^2 + 0.34s + 0.0119} \right) \\ \left( \frac{-s^3 + 0.59s^2 - 0.14s + 0.014}{s^3 + 0.59s^2 + 0.14s + 0.01} \right) & \left( \frac{-s^3 + 0.819s^2 - 0.34s + 0.0119}{s^3 + 0.81s^2 + 0.34s + 0.0119} \right) \end{bmatrix} \quad (9)$$

The function for conversion of transfer function is shown as ratio between two polynomials in factorial form, as it is shown in the equation down below:

$$[z, p, k]^{-1} = tf2zp(num, den)$$

The zeroes and poles are given with:

- z =
- 0.2721 + 0.2400i
- 0.2721 - 0.2400i
- 0.0154
- p =
- 3.1942
- 0.1851 + 0.5763i
- 0.1851 - 0.5763i
- 0.0256

Analyzing the poles of the model of glass capillary, it is obvious that all the poles of the system are negative and they are located in left half plane, so the system is stable.

5. CONCLUSIONS

The question of pole-assignment for glass capillary tube has been considered. A method for pole placement for time delay systems is described and then analyzed in detail. The Padé approximation is used for the system of third order, and the required finite time interval is found. For systems of higher order it might be applied program support in symbolic languages such as Maple. It is explained how to improve dynamic characteristics of the system by using

The transfer function of the system with delay in state, is irrational and has no s polynomial in the numerator and in denominator. In frequency based analysis of control systems [16], it is essential to substitute transcendental term with an approximation in form of rational transfer function, according to the reference [15].

After applying Padé approximation, closed-loop transfer function has the following form:

finite spectrum assignment method for multivariable systems and with Pade approximation eliminate delay. Poles of the system are located at required places and it is shown that system is stable.

PROGRAM SUPPORT

```
#
#   Program support for pole placement
#
#   Definition of procedure pstd
pstd:=proc(input 1)
#   Reading of input parameter values
read input1;
wuth(lunalg):
uu := array(udentuty, 1..2, 1..2);
l := scalarmul(uu, -1);
cd := coldum(e);
rd := rowdum(e);
for u from 1 to cd do
    c[u,j] := solve(denom(e[u,j]));
uf u=j then
    p[u,j] := p[u,j]*1/numer(e[u,j]);
else
    p[u,j] := 0
fu:
r[u,j] := lcoeff(numer(e([u,j]));
l*c[u,j];
    od;
od;
WO := multiply(r,p);
for u from 1 to cd do
    for j from 1 to rd do
```

```

    if u=j then
        gu[u,j] :=scalarmul(e,1);
    else
        gu[u,j] :=0;
    fu;
    od;
od;
# Determination of matrix Q
Q:=det[sl-A0-A1*e^-st];
# Applying Pade approximation
AP=1-k1*s+k2*s^2/1+k1*s+k2*s^2
# Solving the equation
stabilityissue=solve(Q);

```

## NOMENCLATURE

$A, B, C, D$	matrix
$den$	denominator
$G$	transfer function
$k$	system gain
$n$	degree of the polynomial
$num$	numerator
$p$	system pole
$P$	polynomial matrix
$Q$	polynomial matrix
$s$	complex number
$tf$	transfer function
$z$	system zero

## Greek letters

$\tau$  time delay  $\text{sec}^{-1}$

## Subscripts

0 initial transfer function  
 $y_v$  closed loop transfer function

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