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A STUDY OF ACCELERATED LIFE TESTING DESIGN USING GEOMETRIC PROCESS FOR GENERALIZED EXPONENTIAL DISTRIBUTION USING TIME CONSTRAINT

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ABSTRACT
This study deals with the utilization of Generalized Exponential geometric process model for the analysis of accelerated life testing under constant stress using type-I censored data. Assuming that the lifetimes of units under increasing stress levels form a geometric process, the maximum likelihood estimation approach is used for the estimation of parameters and acceleration factor. The confidence intervals (CIs) of the model parameters are derived. The simulation study is conducted to evaluate the performance of the estimators with different pre-fixed parameters.

KEYWORDS: geometric process, generalized Exponential distribution, Maximum Likelihood Estimator, Fisher Information Matrix, Asymptotic Confidence Interval, Simulation Study.

INTRODUCTION
In the modern era there are many problems of life testing which require a long time to acquire the test data at the specified use condition. Therefore, it is becoming more and more difficult to obtain information about lifetime of such products or system with high reliability under usual operating conditions. In such problems, accelerated life tests (ALTs) or partially accelerated life tests (PALTs) are often used to shorten the lives of test items and to reduce the experimental time and the cost incurred in the experiment.

ALT, generally deals with three types of stress loadings: constant stress, step stress and Progressive stress. In a constant stress accelerated test, each unit is run at a pre-specified constant stress level which does not vary with time. Thus every item is run at only one stress level until the item fails or the test is stopped for other reasons. In use, most products such as semiconductors and microelectronics, capacitors, lamps …etc, are subjected to such stress loading pattern. This type of stress is widely used and preferred because it is much easier to apply, widely publicized and empirically verified.

Many researchers in the world have published their work on constant stress accelerated life testing procedure, for example, Ahmad et al. [1], Islam and Ahmad [2], Ahmad and Islam [4], Ahmad et al.[5] and Ahmad [6] discuss the optimal constant stress accelerated life test designs under periodic inspection and Type-I censoring. Yang [7] proposed an optimal design of 4-level constant stress ALT plans considering different censoring times. Pan et al. [8] proposed a bi-variate constant stress accelerated degradation test model by assuming that the copula parameter is a function of the stress level that can be described by the logistic function. Wilkins and johns [9] considered constant stress accelerated life test based on Weibull distribution with constant shape and a log linear link between scale the stress factor which is terminated by a Type-II censoring regime at one of the stress level.

It was first of all Lam [10] who introduced the concept of geometric process in accelerated life testing while analysing the problems of repair replacement. Further Lam [11] studied the geometric process model for a multistate system and concluded a replacement policy to minimize the long run average cost per unit time. Since then a large amount of studies in maintenance problems and system reliability have been shown that a GP model is a good and simple model for analysis of data with a single trend or multiple trends, for example, Lam and Zhang [12], Lam [13] and Zhang [14]. Huang [15] introduced the GP model for the analysis of constant stress ALT with complete and censored exponential samples. Kamal et al. [16] extended the GP model for the analysis of complete Weibull failure data in constant stress ALT. Zhou et al. [17] implemented the GP in ALT based on the progressive Type-I hybrid censored Rayleigh failure data. Kamal et al. [18] used the geometric process for the analysis of constant stress accelerated life testing for Pareto Distribution with complete data. Saxena [19] introduced the Rayleigh geometric process model for the analysis of accelerated life testing under constant stress. Sadia Anwar et al. [20] presented the mathematical model of accelerated life testing for Marshall-Olkin extended exponential distribution using geometric process. Then Sadia Anwar et al. [21] extended her work using type I censored data. Recently Kamal [22] presented an application of geometric process in accelerated life testing analysis on type-I censored Weibull failure data.

In the present study we applied constant stress accelerated life testing for generalized Exponential distribution using geometric process with type-I censored data. The estimates of Parameters are obtained by maximum likelihood estimation technique and Confidence intervals for parameters are obtained by using the asymptotic properties. Lastly statistical properties of estimates and confidence intervals are examined through a simulation study.

THE MODEL

The Geometric Process

A geometric process describes a stochastic process $\{X_n, n = 1,2,...\}$, where there exists a real valued $\upsilon > 0$ such that $\{\upsilon^{n-1} X_n, n = 1,2,...\}$ forms a renewal process. It can be shown that if $\{X_n, n = 1,2,...\}$ is a GP and the probability density function of first order statistics X_1 is $f(x)$ with mean μ and variance σ^2 then the probability density function of nth order statistics X_n will be $\upsilon^{n-1} f(\upsilon^{n-1} x)$ with $E(X_n) = \frac{\mu}{\upsilon^{n-1}}$ and $\text{var}(X_n) = \frac{\sigma^2}{\upsilon^{2(n-1)}}$.

Thus υ, μ and σ^2 are three important parameters of GP.

The Generalized Exponential Distribution

The probability density function (pdf) of a generalized Exponential distribution is given by

$$f(x,\beta,\lambda) = \frac{\beta}{\lambda} e^{-\frac{x}{\lambda}} \left(1 - e^{-\frac{x}{\lambda}}\right)^{\beta-1}, \qquad x > 0$$

Where $\beta > 0$, is the shape parameter and $\lambda > 0$, is the scale parameter of the distribution. Generalized exponential distribution with the shape parameter β and the scale parameter λ will be denoted by $GE(\beta,\lambda)$. $GE(1,\lambda)$, represents the exponential distribution with the scale parameter λ.

It is observed that the two-parameter Generalized Exponential distribution can be used quite electively in analyzing many lifetime data, particularly in place of two-parameter gamma and two-parameter Weibull distributions [3]. Depending on the shape parameter, it can have an increasing and decreasing failure rates. The cumulative distribution function (cdf) of generalized Exponential distribution is

$$F(x,\beta,\lambda) = \left(1 - e^{-\frac{x}{\lambda}}\right)^{\beta}, \qquad \beta,\lambda,x > 0$$

The survival function of the Generalized Exponential distribution takes the following form

$$S(x,\beta,\lambda) = 1 - \left(1 - e^{-\frac{x}{\lambda}}\right)^{\beta}$$

The failure rate or hazard rate is given by

$$h(x,\beta,\lambda)=\frac{\frac{\beta}{\lambda}e^{-\frac{x}{\lambda}}\left(1-e^{-\frac{x}{\lambda}}\right)^{\beta-1}}{1-\left(1-e^{-\frac{x}{\lambda}}\right)^{\beta}}$$

The shape of the hazard function depends only on shape parameter β. It can be observed that the generalized exponential distribution has a log-concave density for $\beta > 1$ and it is log-convex for $\beta \le 1$. Therefore for the fixed value of scale parameter λ, the generalized exponential distribution has an increasing hazard function for $\beta > 1$ and it has a decreasing hazard function for $\beta < 1$. For $\beta = 1$, it has constant hazard function. The hazard function of the generalized exponential distribution behaves exactly the same way as the hazard functions of the gamma distribution, which is quite different from the hazard function of the Weibull distribution [3].

Assumptions and Test Procedure

1. Suppose that an accelerated life test with *s* increasing stress levels in which a random sample of *n* identical items is placed under each stress level and start to operate at the same time. Let x_{ki}, $i=1,2,...,n \quad k=1,2,...,s$ denote observed failure time of *ith* test item under *kth* stress level. Whenever an item fails, it will be removed from the test and the test is terminated at a prespecified censoring time *t* at each stress level and the exact failure times $x_{ki} < t$ of items are observed.

2. The product life under each stress level follows Generalized Exponential distribution denoted by $GE(\beta,\lambda)$ with a constant shape parameter.

3. The scale parameter is a log-linear function of stress. That is, $\log(\lambda_k)=a+bS_k$, where *a* and *b* are unknown parameters depending on the nature of the product and the test method.

4. Let random variables $X_0, X_1, X_2,..., X_S$, denote the lifetimes under each stress level, where X_0 denotes item's lifetime under the design stress at which items will operate ordinarily and sequence $\{X_k, k=1,2,...s\}$ forms a geometric process with ratio $\upsilon > 0$.

The assumption (4) can be shown by the following theorem assuming that there is a log linear relationship between a life and stress (assumption 3).

Theorem: *In ALT, if the stress level is increasing with a constant difference then* the life times of items forms a GP under each stress level. That is, If $S_{k+1}-S_k$ is constant for $k=1,2,...,s-1$, then $\{X_k, k=1,2,...,s\}$ forms a GP.
Proof: From assumption (4), we get

$$\log\left(\frac{\lambda_{k+1}}{\lambda_k}\right)=b(S_{k+1}-S_k)=b\Delta S$$

This shows that the increased stress levels form an arithmetic sequence with a constant difference ΔS.
Now the above equation can be written as

$$\frac{\lambda_{k+1}}{\lambda_k}=e^{b\Delta S}=\upsilon(say) \qquad (1)$$

It is clear from (2.3.1) that

$$\lambda_k=\upsilon\lambda_{k-1}=\upsilon^2\lambda_{k-2}=...=\upsilon^k\lambda$$

The lifetime PDF of an item at the kth stress level is

$$f_{X_k}(x) = \frac{\beta}{\lambda_k} e^{\frac{-x}{\lambda_k}} \left(1 - e^{\frac{-x}{\lambda_k}}\right)^{\beta-1}$$

$$= \frac{\beta \upsilon^k}{\lambda} e^{\frac{-\upsilon^k x}{\lambda}} \left(1 - e^{\frac{-\upsilon^k x}{\lambda}}\right)^{\beta-1} \qquad (2)$$

This implies that

$$f_{X_k}(x) = \upsilon^k f_{X_0}(\upsilon^k x) \qquad (3)$$

From the definition of GP and from expression (3) it is clear that, if density functions of X_0 is $f_{X_0}(x)$, then the pdf of X_k will be given by $\upsilon^k f_{X_0}(\upsilon^k x),\ k = 1,2,...,s$. Therefore, it is clear that lifetimes under a sequence of arithmetically increasing stress levels form a GP with ratio υ.

Now, the pdf of a lifetime of an item at the k^{th} stress level is

$$f_{x_k}(x/\beta,\lambda,\upsilon) = \frac{\beta \upsilon^k}{\lambda} e^{\frac{-\upsilon^k x}{\lambda}} \left(1 - e^{\frac{-\upsilon^k x}{\lambda}}\right)^{\beta-1} \qquad (4)$$

It is clear from above expression that if lifetimes of items under a sequence of increasing stress level form a geometric process with ratio υ and if the life distribution at design stress level is generalized Exponential with characteristic λ, then the life distribution at k^{th} stress level will also be generalized Exponential with characteristic life $\frac{\upsilon^k}{\lambda}$

MAXIMUM LIKELIHOOD ESTIMATION

Let the test at each stress level is terminated at time *t* and only $x_{ki} \le t$ failure times are observed. Assume that $r_k (\le n)$ failures at the kth stress level are observed before the test is suspended and $(n - r_k)$ units are still survived the entire test without failing.

Now the likelihood function for constant stress ALT with Type I censored Generalized Exponential failure data using GP at one of the stress level is given by

$$L_k(\beta,\lambda,\upsilon) = \frac{n!}{(n-r_k)!} \left[\left(\frac{\beta}{\lambda}\upsilon^k\right)^{r_k} \prod_{i=1}^{r_k} \left(1 - e^{-x_{k(i)}\frac{\upsilon^k}{\lambda}}\right)^{\beta-1} \exp\left(-x_{k(i)}\frac{\upsilon^k}{\lambda}\right) \right] \left[1 - \left(1 - e^{-t\frac{\upsilon^k}{\lambda}}\right)^{\beta}\right]^{n-r_k} \qquad (5)$$

Therefore, the likelihood function of observed data for total stress levels is

$$L_k(\beta,\lambda,\upsilon) = L_1 \times L_2 \times ... \times L_s$$

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$$=\prod_{k=1}^{s}\left[\frac{n!}{(n-r_k)!}\left\{\left(\frac{\beta}{\lambda}\upsilon^{k}\right)^{r_k}\prod_{i=1}^{r_k}\left(1-e^{-x_{k(i)}\frac{\upsilon^k}{\lambda}}\right)^{\beta-1}\exp\left(-x_{k(i)}\frac{\upsilon^k}{\lambda}\right)\right\}\left\{1-\left(1-e^{-t\frac{\upsilon^k}{\lambda}}\right)^{\beta}\right\}^{n-r_k}\right] \qquad (6)$$

The log-likelihood function corresponding above expression takes the form

$$l = InL_k(\beta,\lambda,\upsilon)$$

$$=\sum_{k=1}^{s}\left[In\left(\frac{n!}{(n-r_k)!}\right)+r_k In\beta - r_k In\lambda + r_k k In\upsilon + (\beta-1)\sum_{i=1}^{r_k} InA - \frac{\upsilon^k}{\lambda}\sum_{i=1}^{r_k} x_{k(i)} + (n-r_k) InB\right] \qquad (7)$$

Where

$$A = 1-e^{-x_{k(i)}\frac{\upsilon^k}{\lambda}} \text{ And } B = 1-\left(1-e^{-t\frac{\upsilon^k}{\lambda}}\right)^{\beta}$$

MLEs of α,λ and υ are obtained by solving the following normal equations

$$\frac{\partial l}{\partial\beta}=\sum_{k=1}^{s}\left[\frac{r_k}{\beta}+\sum_{i=1}^{r_k} InA-\frac{(n-r_k)}{B}\left(1-e^{-t\frac{\upsilon^k}{\lambda}}\right)^{\beta} In\left(1-e^{-t\frac{\upsilon^k}{\lambda}}\right)\right]=0 \qquad (8)$$

$$\frac{\partial l}{\partial\upsilon}=\sum_{k=1}^{s}\left[\frac{r_k k}{\upsilon}+(\beta-1)k\frac{\upsilon^{k-1}}{\lambda}\sum_{i=1}^{r_k}\frac{x_{k(i)}}{A}e^{-x_{k(i)}\frac{\upsilon^k}{\lambda}} - k\frac{\upsilon^{k-1}}{\lambda}\sum_{k=1}^{r_k} x_{k(i)} - \frac{(n-r_k)}{B}\beta tk\frac{\upsilon^{k-1}}{\lambda}\left(1-e^{-t\frac{\upsilon^k}{\lambda}}\right)^{\beta-1} e^{-t\frac{\upsilon^k}{\lambda}}\right]=0 \qquad (9)$$

$$\frac{\partial l}{\partial\lambda}=\sum_{k=1}^{s}\left[-\frac{r_k}{\lambda}-(\beta-1)\frac{\upsilon^k}{\lambda^2}\sum_{i=1}^{r_k}\frac{x_{k(i)}}{A}e^{-x_{k(i)}\frac{\upsilon^k}{\lambda}} + \frac{\upsilon^k}{\lambda^2}\sum_{i=1}^{r_k} x_{k(i)} + \frac{(n-r_k)}{B}\beta t\frac{\upsilon^k}{\lambda^2}\left(1-e^{-t\frac{\upsilon^k}{\lambda}}\right)^{\beta-1} e^{-t\frac{\upsilon^k}{\lambda}}\right]=0 \qquad (10)$$

Equations (8), (9) and (10) are nonlinear; therefore, it is very difficult to obtain a solution in closed form. So, Newton-Raphson method is used to solve these equations simultaneously to obtain $\hat{\beta},\hat{\upsilon}$ *and* $\hat{\lambda}$.

ASYMTOTIC CONFIDENCE INTERVAL ESTIMATES

The large sample theory states that the ML estimators are consistent and normally distributed under some appropriate regularity conditions. It is impossible to obtain the exact confidence intervals (CIs) because the above estimates of parameters are not in closed form. So instead of exact CIs, asymptotic CIs based on the asymptotic normal distribution of ML estimators are obtained.

The Fisher-information matrix composed of the negative second partial derivatives of log likelihood function can be written as

$$F = \begin{bmatrix} -\frac{\partial^2 l}{\partial \beta^2} & -\frac{\partial^2 l}{\partial \beta \partial \upsilon} & -\frac{\partial^l l}{\partial l \partial \lambda} \\ -\frac{\partial^2 l}{\partial \upsilon \partial \beta} & -\frac{\partial^2 l}{\partial \upsilon^2} & -\frac{\partial^2 l}{\partial \upsilon \partial \lambda} \\ -\frac{\partial^2 l}{\partial \lambda \partial \beta} & -\frac{\partial^2 l}{\partial \lambda \partial \upsilon} & -\frac{\partial^2 l}{\partial \lambda^2} \end{bmatrix}$$

The elements of above fisher matrix are given as;

$$\frac{\partial^2 l}{\partial \beta^2} = \sum_{k=1}^{s}\left[-\frac{r_k}{\beta^2} - \frac{(n-r_k)}{B^2}(InC)^2\left(B + C^\beta\right)C^\beta\right] \tag{11}$$

$$\frac{\partial^2 l}{\partial \upsilon^2} = \sum_{k=1}^{s}\left[-\frac{r_k k}{\upsilon^2} + \frac{(\beta-1)k\upsilon^{k-2}}{\lambda}\left\{\frac{k\upsilon}{\lambda}\sum_{i=1}^{r_k} x_{k(i)}^2\left(\frac{D^2 - AD}{A^2}\right) + \sum_{i=1}^{r_k} x_{k(i)}(k-1)\frac{D}{A}\right\} - \frac{k(k-1)\upsilon^{k-2}}{\lambda}\sum_{k=1}^{r_k} x_{k(i)}\right]$$

$$+\sum_{k=1}^{s}\left[\beta(n-r_k)\frac{tk}{\lambda B}EC^{\beta-1}\left\{\frac{tk}{\lambda}\upsilon^{2(k-1)} - \frac{tk}{\lambda C}(\beta-1)E\upsilon^{2(k-1)} - (k-1)\upsilon^{k-2} - \frac{\beta tk}{\lambda B}E\upsilon^{2(k-1)}C^{\beta-1}\right\}\right] \tag{12}$$

$$\frac{\partial^2 l}{\partial \lambda^2} = \sum_{k=1}^{s}\left[\frac{r_k}{\lambda^2} - \frac{\beta-1}{\lambda^3}\upsilon^k\left\{\frac{\upsilon^k}{\lambda}\sum_{i=1}^{r_k} x_{k(i)}^2 D\frac{A+D}{A^2} + 2\sum_{i-1}^{r_k} x_{k(i)}\frac{D}{A}\right\} - 2\frac{\upsilon^k}{\lambda^3}\sum_{i=1}^{r_k} x_{k(i)}\right]$$

$$+\sum_{i=1}^{s}\left[\beta t\upsilon^k(n-r_k)C^{\beta-1}\frac{E}{B}\left\{\frac{t\upsilon^k}{\lambda^4}\left(1 - E\frac{\beta-1}{C} - \beta C^{\beta-1}\frac{E}{B}\right) - \frac{2}{\lambda^3}\right\}\right] \tag{13}$$

$$\frac{\partial^2 l}{\partial \beta \partial \upsilon} = \frac{\partial^2 l}{\partial \upsilon \partial \beta} = \sum_{k=1}^{s}\left[\frac{k\upsilon^{k-1}}{\lambda}\sum_{i=1}^{r_k} x_{k(i)}\frac{D}{A} - \frac{(n-r_k)}{\lambda}Etk\upsilon^{k-1}\frac{C^{\beta-1}}{B^2}\left\{B(1+\beta InC) + \beta C^\beta InC\right\}\right] \tag{14}$$

$$\frac{\partial^2 l}{\partial \beta \partial \lambda} = \frac{\partial^2 l}{\partial \lambda \partial \beta} = \sum_{i=1}^{s}\left[-\frac{\upsilon^k}{\lambda^2}\sum_{i=1}^{r_k} x_{k(i)}\frac{D}{A} + \frac{(n-r_k)}{\lambda^2}Et\upsilon^k\frac{C^{\beta-1}}{B^2}\left\{B + \beta InC\left(B + C^\beta\right)\right\}\right] \tag{15}$$

$$\frac{\partial^2 l}{\partial \upsilon \partial \lambda} = \frac{\partial^2 l}{\partial \lambda \partial \upsilon} = \sum_{k=1}^{s}\left[\frac{(\beta-1)}{\lambda^2}k\upsilon^{k-1}\left\{\sum_{i=1}^{r_k} x_{k(i)}\left(\frac{x_{k(i)}\upsilon^k}{\lambda} - 1\right)\frac{D}{A} + \upsilon^k\sum_{i=1}^{r_k} x_{k(i)}^2\left(\frac{D}{A}\right)^2\right\} + \frac{k\upsilon^{k-1}}{\lambda^2}\sum_{i=1}^{r_k} x_{k(i)}\right]$$

$$-\sum_{i=1}^{s}\left[\frac{\beta(n-r_k)}{\lambda^2}\frac{E}{B^2}tk\upsilon^{k-1}C^{\beta-1}\left\{\left(B-EB\frac{\beta-1}{C}-\beta EC^{\beta-1}\right)\frac{t\upsilon^k}{\lambda}-1\right\}\right] \quad (16)$$

Where,

$$D=e^{-x_{k(i)}\frac{\upsilon^k}{\lambda}},\quad A=1-D,\quad E=e^{t\frac{\upsilon^k}{\lambda}},\quad C=1-E \;\; and \;\; B=1-C^{\beta}$$

The variance covariance matrix can be written as

$$W=\begin{bmatrix}-\frac{\partial^2 l}{\partial\beta^2} & -\frac{\partial^2 l}{\partial\beta\partial\upsilon} & -\frac{\partial^l l}{\partial\beta\partial\lambda}\\ -\frac{\partial^2 l}{\partial\upsilon\partial\beta} & -\frac{\partial^2 l}{\partial\upsilon^2} & -\frac{\partial^2 l}{\partial\upsilon\partial\lambda}\\ -\frac{\partial^2 l}{\partial\lambda\partial\beta} & -\frac{\partial^2 l}{\partial\lambda\partial\upsilon} & -\frac{\partial^2 l}{\partial\lambda^2}\end{bmatrix}^{-1}=\begin{bmatrix}A\mathrm{var}(\hat\beta) & A\mathrm{cov}(\hat\beta\hat\upsilon) & A\mathrm{cov}(\hat\beta\hat\lambda)\\ A\mathrm{cov}(\hat\upsilon\hat\beta) & A\mathrm{var}(\hat\upsilon) & A\mathrm{cov}(\hat\upsilon\hat\lambda)\\ A\mathrm{cov}(\hat\lambda\hat\beta) & A\mathrm{cov}(\hat\lambda\hat\upsilon) & A\mathrm{var}(\hat\lambda)\end{bmatrix}$$

The 100(1-γ) % asymptotic confidence interval for β, υ and λ are then given respectively as

$$\hat\beta\pm Z_{1-\frac{\gamma}{2}}\sqrt{A\mathrm{var}(\hat\beta)},\quad \hat\upsilon\pm Z_{1-\frac{\gamma}{2}}\sqrt{A\mathrm{var}(\hat\upsilon)}\;\; and \;\; \hat\lambda\pm Z_{1-\frac{\gamma}{2}}\sqrt{\mathrm{var}(\hat\lambda)}$$

SIMULATION STADIES

Simulation studies have been performed using R software for illustrating the theoretical results of estimation problem. The performance of the resulting estimators of the acceleration factor and other parameters has been considered in terms of their absolute relative bias (RABias), mean square error (MSE), and relative error (RE).
The Simulation procedures are described below:

Step 1. 1000 random samples of sizes 20, 40, 60, 80 and 100 are generated from the generalized exponential distribution. The data generation from the generalized exponential distribution is very simple, if U has a uniform (0, 1) random number, and then $Y=[\lambda.\log(1-u^{\frac{1}{beta}})/\nu\hat{\ }k]$ follows a generalized exponential distribution. The true parameter values are selected as (β=1.2, $\lambda=2.9$, ν= $1.1, s=4$) and ($\beta=1.2,\ \lambda=2.9, \nu=1.1, s=6$).

Step 2. Censored time t at the normal condition is chosen to be t=4.

Step 3. The parameters are obtained for generalized exponential distribution using geometric process under type I censored sample, for different samples and for each set of true parameters.

Step 4. The optim function is used to estimate the parameters. Further the SEs, RABias, MSEs, and REs of the estimators of scale and shape parameters for all sample sizes and for two sets of parameters are tabulated.

Step 5. The confidence limits with confidence level γ=0.95 and γ =0.99 of the scale and shape parameters are constructed.

Simulation results are summarized in Tables 1 and Table 2. Table 1 gives the estimates of the parameters, SEs, RABias, MSEs, REs and confidence intervals at 95% and 99% of the estimators for the first set ($\beta=1.2,\ \lambda=2.9, \nu=$

$1.1, s = 4$). Same characteristics of the estimates are also obtained in Table 2 for second set i.e.($\beta = 1.2,\ \lambda = 2.9, \nu = 1.1, s = 6$).

From the tables, the following observations can be made on the performance of SS-PALT parameter estimation of the Burr type XII lifetime distribution:

1. The values of parameters and numbers of the stress levels are chosen to be β=1.2, λ=2.9, ν=1.1 and s=4 or 6.
2. For the second set of parameters (β=1.2, $\lambda = 2.9, \nu = 1.1$, s=6), the maximum likelihood estimators have good statistical properties than the first set of parameters (β=1.2, $\lambda = 2.9$, $\nu = 1.1$, s=4) for all sample sizes (see Table 1 and Table 2).

Table: 1 Simulation results based on the initial values for β=1.2, λ=2.9, ν=1.1 and s=4

Sample	Estimate	Mean	SE	RAB	√MSE	RE	LCL	UCL
20	**β**	1.2003	0.1337	0.0226	0.1292	0.1176	0.9381 0.8552	1.4624 1.5453
	λ	3.0959	0.2523	0.0675	0.2437	0.0840	2.6014 2.4450	3.5905 3.7469
	θ	1.000	0.1035	0.0909	0.0999	0.0908	0.7971 0.7329	1.2028 1.2670
40	**β**	1.2205	0.1405	0.0171	0.1357	0.1131	0.9451 0.8580	1.4959 1.5830
	λ	3.0678	0.2487	0.0578	0.2403	0.0828	2.5802 2.4260	3.5554 3.7096
	θ	0.9998	0.1034	0.0908	0.0999	0.0908	0.7972 0.7330	1.2028 1.2670
60	**β**	1.2124	0.1306	0.0103	0.1261	0.1051	0.9564 0.8754	1.4683 1.5493
	λ	3.1068	0.2423	0.0713	0.2340	0.0807	2.6319 2.4816	3.5817 3.7319
	θ	0.9999	0.1035	0.0909	0.1000	0.0909	0.7970 0.7328	1.2029 1.2670
80	**β**	1.2233	0.0647	0.0194	0.0596	0.0496	1.1023 1.0640	1.3443 1.3825
	λ	3.1287	0.2478	0.0788	0.2394	0.0805	2.6428 2.4891	3.6145 3.7682
	θ	1.001	0.1034	0.09089	0.0999	0.09089	0.7971 0.7330	1.2028 1.2670
100	**β**	1.2134	0.0645	0.0111	0.0623	0.0519	1.0868 1.0468	1.3399 1.3799
	λ	3.1277	0.2447	0.0785	0.2364	0.0815	2.6480 2.4962	3.6075 3.7593
	θ	0.9999	0.1035	0.09094	0.10004	0.09094	0.7969 0.7327	1.2029 1.2671

Table: 2 Simulation results based on the initial values for β=1.2, λ=2.9, ν=1.1 and s=6

Sample	Estimate	Mean	SE	RAB	√MSE	RE	LCL	UCL
20	**β**	1.2170	0.0843	0.0141	0.0815	0.0679	1.0516 0.9992	1.3825 1.4347
	λ	3.035	0.1570	0.0465	0.1517	0.0523	2.7272 2.6299	3.3428 3.4402
	θ	0.9999	0.1035	0.0909	0.10001	0.09091	0.7970 0.7329	1.2028 1.2670

40	**β**	1.2269	0.1254	0.0223	0.1211	0.1009	0.9810 0.9033	1.4726 1.5503
	λ	3.0547	0.1816	0.0533	0.1755	0.0605	2.6986 2.5859	3.4108 3.5234
	θ	1.000	0.1034	0.09089	0.09998	0.09089	0.7971 0.733	1.2028 1.267
60	**β**	1.1958	0.1128	0.0034	0.1090	0.0908	0.9746 0.9046	1.4170 1.4869
	λ	3.1126	0.2391	0.0733	0.2310	0.0796	2.6438 2.4955	3.5813 3.7296
	θ	0.99889	0.1035	0.0909	0.09999	0.0909	0.7971 0.7329	1.2028 1.2670
80	**β**	1.2132	0.0919	0.0109	0.0888	0.0740	1.0329 0.9759	1.3934 1.4505
	λ	3.1209	0.2357	0.0761	0.2277	0.0785	2.6588 2.5126	3.5830 3.7292
	θ	0.9999	0.1035	0.09091	0.10001	0.09091	0.7971 0.7329	1.2029 1.2670
100	**β**	1.1859	0.08069	0.0117	0.0779	0.0649	1.0277 0.9776	1.3440 1.3940
	λ	3.1340	0.2470	0.0806	0.2387	0.0823	2.6497 2.4965	3.6183 3.7715
	θ	1.000	0.1035	0.0909	0.0999	0.0909	0.7971 0.7329	1.2028 1.2670

DISCUSSION AND CONCLUSION

This study deals with the analysis of constant stress ALT plan using geometric process model for generalized exponential distribution with complete data. The SEs, MSEs, REs and RABs of the model parameters and acceleration factor were obtained. Based on the asymptotic normality, the coverage rate of 95% and 99% confidence intervals of the model parameters and acceleration factor were also obtained. The simulation study in the table 1 and table 2 above, states that the estimates obtained are very close to the true values of the parameters and are also quite well with relatively small mean squared errors. In the whole study, the parameters are estimated for different cases and it is found that as the sample size and stress level increases, the MSE gets smaller. It implies that a larger sample size and stress level results in a better large sample approximation. Also, the interval estimate of estimators at γ=0.99 is greater than the interval estimators at γ=0.95. Hence, it can be said that the proposed model works quite well in the analysis of accelerated life testing analysis.

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