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LS-SASAKIAN MANIFOLDS WITH SEMI-SYMMETRIC METRIC CONNECTION

L K Pandey
D S Institute of Technology & Management, Ghaziabad (U.P.) 201007, India

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ABSTRACT

The purpose of this paper is to study Lorentzian special Sasakian manifolds and generalized Lorentzian Co-symplectic manifolds [1] with semi-symmetric metric connection [3].

KEYWORDS: Nearly and almost LS-Sasakian manifolds, generalized L-Co-symplectic manifolds, semi-symmetric metric connection, Nijenhuis tensor [2].

INTRODUCTION

An n-dimensional differentiable manifold M_n, on which there are defined a tensor field $F$of type (1, 1), a vector field T, a 1-form A and a Lorentzian metric g, satisfying for arbitrary vector fields $X, Y, Z, \dots$

(1.1) $\overline{\overline{X}} = -X - A(X)T, \quad \overline{T} = 0, \quad A(T) = -1, \quad \overline{X} \overset{\text{def}}{=} FX, \quad A(\overline{X}) = 0, \quad \text{rank } F = n-1.$

(1.2) $g(\overline{X}, \overline{Y}) = g(X,Y) + A(X)A(Y)$, where $A(X) = g(X,T)$,

$`F(X,Y) \overset{\text{def}}{=} g(\overline{X}, Y) = -`F(Y,X)$,

Then M_n is called a Lorentzian contact manifold (an L-Contact manifold).

Let D be a Riemannian connection on M_n, then

An L-Contact manifold is called a Lorentzian special Sasakian manifold (an LS-Sasakian manifold), if

(1.3) (a) $(D_X F)(Y) + A(Y)\overline{X} - `F(X,Y)T = 0 \Leftrightarrow (D_X `F)(Y,Z) - A(Y)`F(Z,X) - A(Z)`F(X,Y) = 0$

(b) $D_X T = \overline{\overline{X}}$

An L-Contact manifold is called a nearly Lorentzian special Sasakian manifold (a nearly LS-Sasakian manifold), if

(1.4) $(D_X `F)(Y,Z) - A(Y)`F(Z,X) - A(Z)`F(X,Y)$
$= (D_Y `F)(Z,X) - A(Z)`F(X,Y) - A(X)`F(Y,Z)$
$= (D_Z `F)(X,Y) - A(X)`F(Y,Z) - A(Y)`F(Z,X)$

An L-Contact manifold is called an almost Lorentzian special Sasakian manifold (an almost LS-Sasakian manifold), if

(1.5) $(D_X `F)(Y,Z) + (D_Y `F)(Z,X) + (D_Z `F)(X,Y) - 2\{A(X)`F(Y,Z) + A(Y)`F(Z,X) + A(Z)`F(X,Y)\} = 0$

An L-Contact manifold is called a generalized Lorentzian Co-symplectic manifold (a generalized L-Co-symplectic manifold), if

(1.6) (a) $(D_X F)Y - A(Y)\overline{D_X T} - (D_X A)(\overline{Y})T = 0 \Leftrightarrow$

(b) $(D_X `F)(Y,Z) + A(Y)(D_X A)(\overline{Z}) - A(Z)(D_X A)(\overline{Y}) = 0$

An L-Contact manifold is called a generalized nearly Lorentzian Co-symplectic manifold (a generalized nearly L-Co-symplectic manifold), if

(1.7) $(D_X `F)(Y,Z) + A(Y)(D_X A)(\overline{Z}) - A(Z)(D_X A)(\overline{Y})$
$= (D_Y `F)(Z,X) + A(Z)(D_Y A)(\overline{X}) - A(X)(D_Y A)(\overline{Z})$
$= (D_Z `F)(X,Y) + A(X)(D_Z A)(\overline{Y}) - A(Y)(D_Z A)(\overline{X})$

An L-Contact manifold is called a generalized almost L-Co-symplectic manifold, if

(1.8) $(D_X `F)(Y,Z) + (D_Y `F)(Z,X) + (D_Z `F)(X,Y) - A(X)\{(D_Y A)(\overline{Z}) - (D_Z A)(\overline{Y})\}$
$-A(Y)\{(D_Z A)(\overline{X}) - (D_X A)(\overline{Z})\} - A(Z)\{(D_X A)(\overline{Y}) - (D_Y A)(\overline{X})\} = 0$

SEMI-SYMMETRIC METRIC CONNECTION

Let us consider a connection B on M_n , defined by

(2.1) $B_X Y \overset{\text{def}}{=} D_X Y + A(Y)X - g(X,Y)T$

The torsion tensor S of B is given by

(2.2) $S(X,Y) = A(Y)X - A(X)Y$

Further, if $(B_X g) = 0$, then B is called a semi-symmetric metric connection.

Put

(2.3) (a) $B_X Y = D_X Y + H(X,Y)$

Where H is a tensor field of type (1, 2), then

(b) $H(X,Y) = A(Y)X - g(X,Y)T$

(c) $`H(X,Y,Z) = A(Y)g(Z,X) - A(Z)g(X,Y)$

(d) $`S(X,Y,Z) = `H(X,Y,Z) - `H(Y,X,Z)$

Where

$`H(X,Y,Z) \overset{\text{def}}{=} g(H(X,Y),Z)$ and $`S(X,Y,Z) \overset{\text{def}}{=} g(S(X,Y),Z)$

In an L-Contact manifold, we have

(2.4) $(B_X `F)(\bar{Y}, \bar{Z}) + (B_X `F)(Y,Z) + A(Y)(B_X A)(\bar{Z}) - A(Z)(B_X A)(\bar{Y}) = 0$

Therefore,

An L-Contact manifold is called an LS-Sasakian manifold, if

(2.5) (a) $(B_X `F)(Y,Z) - 2A(Y)`F(Z,X) - 2A(Z)`F(X,Y) = 0$

(b) $B_X T = 2\,\bar{\bar{X}}$

On this manifold, we have

(2.6) (a) $(B_X A)(\bar{Y}) = 2`F(X,Y)$ $\Leftrightarrow$ (b) $(B_X A)(Y) = -2g(\bar{X}, \bar{Y})$

An L-contact manifold is called a nearly LS-Sasakian manifold, if

(2.7) $(B_X `F)(Y,Z) - 2A(Y)`F(Z,X) - 2A(Z)`F(X,Y)$
$= (B_Y `F)(Z,X) - 2A(Z)`F(X,Y) - 2A(X)`F(Y,Z)$
$= (B_Z `F)(X,Y) - 2A(X)`F(Y,Z) - 2A(Y)`F(Z,X)$

The equation of a nearly LS-Sasakian manifold can also be written as

(2.8) (a) $(B_X F)Y + (B_Y F)X + 2A(Y)\bar{X} + 2A(X)\bar{Y} = 0 \Leftrightarrow$

(b) $(B_X `F)(Y,Z) - (B_Y `F)(Z,X) - 2A(Y)\,`F(Z,X) + 2A(X)\,`F(Y,Z) = 0$

These equations can be modified as

(2.9) (a) $(B_X F)\bar{Y} + (B_{\bar{Y}} F)X + 2A(X)\bar{\bar{Y}} = 0 \Leftrightarrow$

(b) $(B_X `F)(\bar{Y}, Z) - (B_{\bar{Y}} `F)(Z,X) - 2A(X)g(\bar{Y}, \bar{Z}) = 0$

(2.10) (a) $(B_X F)\bar{\bar{Y}} + (B_{\bar{\bar{Y}}} F)X - 2A(X)\bar{Y} = 0 \Leftrightarrow$

(b) $(B_X `F)\left(\bar{\bar{Y}}, Z\right) - \left(B_{\bar{\bar{Y}}} `F\right)(Z,X) - 2A(X)\,`F(Y,Z) = 0$

(2.11) (a) $(B_X F)Y + (B_Y F)X - A(Y)\{\overline{B_X T} - (B_T F)X\} - A(X)\{\overline{B_Y T} - (B_T F)Y\} = 0 \Leftrightarrow$

(b) $(B_X `F)(Y,Z) - (B_Y `F)(Z,X) + A(Y)\{(B_X A)(\bar{Z}) - (B_T `F)(Z,X)\} + A(X)\,\{(B_Y A)(\bar{Z}) - (B_T `F)(Z,Y)\} = 0$

An L-contact manifold is called an almost LS-Sasakian manifold, if

(2.12) (a) $(B_X `F)(Y,Z) + (B_Y `F)(Z,X) + (B_Z `F)(X,Y) - 4\{A(X)\;`F(Y,Z) + A(Y)\,`F(Z,X) + A(Z)\,`F(X,Y)\} = 0$

This gives

(b) $(B_{\bar{X}} `F)(\bar{Y}, \bar{Z}) + (B_{\bar{Y}} `F)(\bar{Z}, \bar{X}) + (B_{\bar{Z}} `F)(\bar{X}, \bar{Y}) = 0$

An L-Contact manifold is called a generalized L-Co-symplectic manifold, if

(2.13) (a) $(B_X `F)(Y,Z) + A(Y)(B_X A)(\bar{Z}) - A(Z)(B_X A)(\bar{Y}) = 0$

This gives

(b) $(B_X `F)(\bar{Y}, \bar{Z}) = 0$

An L-Contact manifold is called a generalised nearly L-Cosymplectic manifold, if

(2.14) (a) $(B_X\,`F)(Y,Z)+A(Y)(B_XA)(\overline{Z})-A(Z)(B_XA)(\overline{Y})$
$=(B_Y\,`F)(Z,X)+A(Z)(B_YA)(\overline{X})-A(X)(B_YA)(\overline{Z})$
$=(B_Z\,`F)(X,Y)+A(X)(B_ZA)(\overline{Y})-A(Y)(B_ZA)(\overline{X})$

This gives

(b) $(B_{\overline{X}}\,`F)(\overline{Y},\ \overline{Z})=(B_{\overline{Y}}\,`F)(\overline{Z},\ \overline{X})=(B_{\overline{Z}}\,`F)(\overline{X},\ \overline{Y})$

An L-Contact manifold is called a generalized almost L-Co-symplectic manifold, if

(2.15) (a) $(B_X\,`F)(Y,Z)+(B_Y\,`F)(Z,X)+(B_Z\,`F)(X,Y)-A(X)\{(B_YA)(\overline{Z})-(B_ZA)(\overline{Y})\}$
$-A(Y)\{(B_ZA)(\overline{X})-(B_XA)(\overline{Z})\}-A(Z)\{(B_XA)(\overline{Y})-(B_YA)(\overline{X})\}=0$

Which implies

(b) $(B_{\overline{X}}\,`F)(\overline{Y},\ \overline{Z})+(B_{\overline{Y}}\,`F)(\overline{Z},\ \overline{X})+(B_{\overline{Z}}\,`F)(\overline{X},\ \overline{Y})=0$

PROPERTIES

From (2.5) (a), we see that in an LS $-$ Sasakian manifold, $B_TF=0$. We will now consider nearly LS-Sasakian manifold

Putting T for X in (2.7), we get

(3.1) $(B_T`F)(Y,Z)=-(B_YA)(\overline{Z})+2`F(Y,Z)=(B_ZA)(\overline{Y})+2`F(Y,Z)$

Hence

(3.2) (a) $(B_YA)(\overline{Z})+(B_ZA)(\overline{Y})=0\ \Leftrightarrow$ (b) $B_TT=0$

Barring Y and Z in equation (3.1) and then using (2.4) and (3.2), we get

(3.3) $(B_T`F)(Y,Z)=-(B_{\overline{Y}}A)(Z)-2`F(Y,Z)=(B_{\overline{Z}}A)(Y)-2`F(Y,Z)$

From (3.1) and (3.3), we obtain

(3.4) (a) $(B_{\overline{Y}}A)(Z)+(B_ZA)(\overline{Y})=-4`F(Y,Z)$ (b) $(B_YA)(Z)+(B_ZA)(Y)=-4g(\overline{Y},\ \overline{Z})$

Hence, on a nearly LS-Sasakian manifold, (3.1), (3.2), (3.3) and (3.4) hold.

Almost LS-Sasakian manifold will now be considered. Putting T for X in (2.12) (a), we get

(3.5) (a) $(B_T`F)(Y,Z)=(B_YA)(\overline{Z})-(B_ZA)(\overline{Y})-4`F(Y,Z)\ \Leftrightarrow$ (b) $B_TT=0$

Barring Y and Z in equation (3.5) (a) and using (2.4), we get

(3.6) $(B_T`F)(Y,Z)=(B_{\overline{Y}}A)(Z)-(B_{\overline{Z}}A)(Y)+4`F(Y,Z)$

From (3.5) (a) and (3.6), we obtain

(3.7) (a) $(B_{\overline{Y}}A)(Z)-(B_YA)(\overline{Z})-(B_{\overline{Z}}A)(Y)+(B_ZA)(\overline{Y})+8`F(Y,Z)=0\Leftrightarrow$
(b) $(B_{\overline{Y}}A)(\overline{Z})+(B_{\overline{Z}}A)(\overline{Y})+(B_YA)(Z)+(B_ZA)(Y)+8g(\overline{Y},\ \overline{Z})=0$

from (2.7) and (2.14) (a), we see that

A nearly LS-Sasakian manifold is a generalized nearly L-Co-symplectic manifold, if

(3.8) (a) $(B_XA)(\overline{Y})=2`F(\overline{X},\ \overline{Y})\Leftrightarrow$ (b) $(B_XA)(Y)=-2g(\overline{X},\ \overline{Y})\Leftrightarrow$ (c) $B_XT=2\overline{\overline{X}}$

Also, Making the use of (2.12) (a) and (2.15) (a), we see that

A generalized almost L-Co-symplectic manifold is an almost LS-Sasakian manifold, if

(3.9) $(B_XA)(\overline{Y})-(B_YA)(\overline{X})=4`F(X,Y)$

NIJENHUIS TENSOR

In an L-Contact manifold with the semi-symmetric metric connection B, Nijenhuis tensor is given by

(4.1) $`N(X,Y,Z)=(B_{\overline{X}}`F)(Y,Z)+(B_{\overline{Y}}`F)(Z,X)+(B_X`F)(Y,\overline{Z})+(B_Y`F)(\overline{Z},X)$

Where

$$`N(X,Y,Z)\overset{\text{def}}{=}g(N(X,Y),Z)$$

Barring X,Y,Z in (4.1) and using equations (2.7), we see that a nearly LS-Sasakian manifold is completely integrable, if

(4.2) $(B_{\overline{X}}`F)(\overline{Y},\overline{\overline{Z}})+(B_{\overline{Y}}`F)(\overline{Z},\overline{\overline{X}})=0.$

Barring X, Y, Z in (4.1) and using equations (2.12) (b), we can prove that an almost LS-Sasakian manifold is completely integrable, if

(4.3) $(B_{\overline{Z}}`F)(\overline{X},\overline{\overline{Y}}) = 0.$

INDUCED CONNECTION IN AN LS-SASAKIAN MANIFOLD

Let M_{2m-1} be submanifold of M_{2m+1} and let $c : M_{2m-1} \to M_{2m+1}$ be the inclusion map such that

$$d \in M_{2m-1} \to cd \in M_{2m+1} ,$$

Where c induces a linear transformation (Jacobian map) $J : T'_{2m-1} \to T'_{2m+1}$.

T'_{2m-1} is a tangent space to M_{2m-1} at point d and T'_{2m+1} is a tangent space to M_{2m+1} at point cd such that

$$\hat{X} \text{ in } M_{2m-1} \text{ at } d \to J\hat{X} \text{ in } M_{2m+1} \text{ at } cd$$

Let $\tilde{g}$ be the induced Lorentzian metric in M_{2m-1} . Then we have

(5.1) $\tilde{g}(\hat{X},\hat{Y}) = ((g(J\hat{X},J\hat{Y}))b$

We now suppose that a semi-symmetric metric connection B in an LS-Sasakian manifold is given by

(5.2) $B_X Y = D_X Y + A(Y)X - g(X,Y)T$,

Where X and Y are arbitrary vector fields of M_{2m+1}. If

(5.3) $T_1 = Jt_1 - \rho_1 M - \sigma_1 N$

Where t_1 is C^∞ vector fields in M_{2m-1} and M and N are unit normal vectors to M_{2m-1}.

Denoting by $\grave{D}$ the connection induced on the submanifold from D, Let

(5.4) $D_{JX} J\hat{Y} = J(\grave{D}_X\hat{Y}) - h(\hat{X},\hat{Y})M - k(\hat{X},\hat{Y})N$

Where h and k are symmetric bilinear functions in M_{2m-1}. Similarly we have

(5.5) $B_{JX} J\hat{Y} = J(\grave{B}_X\hat{Y}) - p(\hat{X},\hat{Y})M - q(\hat{X},\hat{Y})N$,

Where $\grave{B}$ is the connection induced on the submanifold from B and p, q are symmetric bilinear functions in M_{2m-1}

Inconsequence of (5.2), we have

(5.6) $B_{JX} J\hat{Y} = D_{JX} J\hat{Y} + A(J\hat{Y})J\hat{X} - g(J\hat{X},J\hat{Y})T_1$

Using (5.4), (5.5) and (5.6), we get

(5.7) $J(\grave{B}_X\hat{Y}) - p(\hat{X},\hat{Y})M - q(\hat{X},\hat{Y})N = J(\grave{D}_X\hat{Y}) - h(\hat{X},\hat{Y})M - k(\hat{X},\hat{Y})N + A_1(J\hat{Y})J\hat{X} - g(J\hat{X},J\hat{Y})T_1$

Using (5.3), we obtain

(5.8) $J(\grave{B}_X\hat{Y}) - p(\hat{X},\hat{Y})M - q(\hat{X},\hat{Y})N = J(\grave{D}_X\hat{Y}) - h(\hat{X},\hat{Y})M - k(\hat{X},\hat{Y})N + a_1(\hat{Y})J\hat{X} - \tilde{g}(\hat{X},\hat{Y})(Jt_1 - \rho_1 M - \sigma_1 N)$

Where $\tilde{g}(\hat{Y},t_1) \stackrel{\text{def}}{=} a_1(\hat{Y})$

This implies

(5.9) $\grave{B}_X\hat{Y} = \grave{D}_X\hat{Y} + a_1(\hat{Y})\hat{X} - \tilde{g}(\hat{X},\hat{Y})t_1$

Iff

(5.10) $\tilde{g}(\hat{X},\hat{Y}) = \frac{1}{\rho_1}\{h(\hat{X},\hat{Y}) - p(\hat{X},\hat{Y})\} = \frac{1}{\sigma_1}\{k(\hat{X},\hat{Y}) - q(\hat{X},\hat{Y})\}$

Therefore,

Theorem 5.1 The connection induced on a submanifold of an LS-Sasakian manifold with a semi-symmetric metric connection with respect to unit normal vectors M and N is also semi-symmetric metric connection iff (5.10) holds.

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