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CHEMICAL REACTION AND THERMAL RADIATION EFFECTS ON UNSTEADY MHD FLOW OF AN INCOMPRESSIBLE VISCOUS FLUID PAST A MOVING VERTICAL CYLINDER

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ABSTRACT

In the present study, the effects of chemical reaction and thermal radiation on unsteady MHD flow of a viscous, electrically conducting and incompressible fluid mixture past a moving vertical cylinder is studied. The fluid is a gray, absorbing-emitting but non scattering medium and the Rosseland approximation is used to describe the radiative heat flux in the energy equation. The governing dimensionless coupled non-linear partial differential equations are solved numerically using finite difference method (FDM) by developing suitable codes in MATLAB. The fluid velocity, temperature and species concentration profiles have been drawn for time and various flow parameters such as Radiation parameter (N) and chemical reaction parameter ( ), and results are discussed.

KEYWORDS: Viscous fluid, Unsteady flow, First order Chemical reaction, Thermal radiation, Finite-difference scheme, Moving vertical cylinder.

INTRODUCTION

The study of flow problems, which involve the interaction of several phenomena, has a wide range of applications in the field of science and technology. One such study is related to the effects of free convection MHD flow, which plays an important role in agriculture, engineering and petroleum industries. The problem of free convection under the influence of magnetic field has attracted the interest of many researchers in view of its application in geophysics and astrophysics.

An experimental and analytical study is reported by Evas *et al*., [1] for transient natural convection in a vertical cylinder. The effects of heat and mass transfer on natural convection flow over a vertical cylinder were studied by Chen and Yuh [2]. Combined heat and mass transfer effects on moving vertical cylinder that of steady and unsteady flow were analyzed by Takhar *et al*., [3] and Ganesan and Loganathan [4]. They were using an implicit finite-difference scheme of Crank-Nicolson type. A numerical solution for the transient natural convection flow over a vertical cylinder under the combined buoyancy effect of heat and mass transfer was given by Ganesan and Rani [5]. An implicit finite-difference scheme is used to solve the problem. Shanker and Kishan [6] presented the effect of mass transfer on the MHD flow past an impulsively started infinite vertical plate. Ganesan and Rani [7] studied the effect of MHD on unsteady free convection flow past a vertical cylinder with heat and mass transfer. Magnetic field effect on a moving vertical cylinder with constant heat flux was given by Ganesan and Loganathan [8]. In the context of space technology and in processes involving high temperatures, the effects of radiation are of very important. Studies with interaction of thermal radiation and free convection were Arpaci [9], Cess [10], Cheng and Ozisik [11], Raptis [12], Hossain and Takhar [13, 14]. In all these papers, the flow is considered steady. The unsteady flow past a moving plate in the presence of free convection and radiation were studied by Monsour [15] and Raptis and Perdikis [16]. Sharma and Dutta [17] have analyzed chemical reaction and thermal radiation effects on unsteady MHD flow over an infinite vertical oscillating porous plate with heat source. Radiation and mass transfer effects on flow of an incompressible viscous fluid past a moving vertical cylinder was studied by Ganesan and loganathan [18]. Chambre and Young [19] have analyzed a first order chemical reaction in the neighbourhood of a horizontal plate.

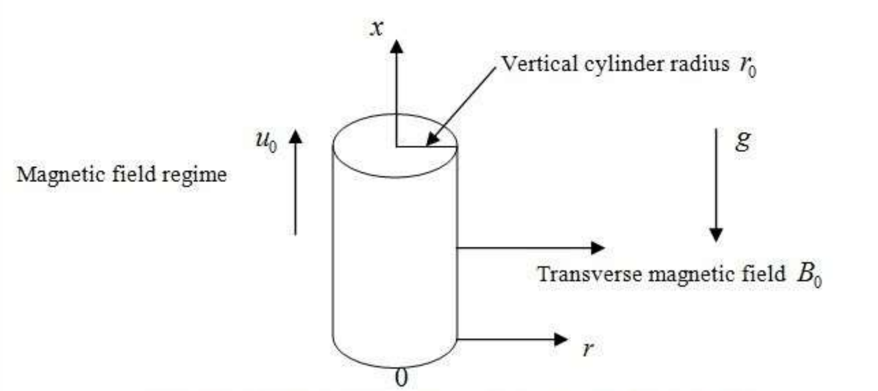
The study of heat and mass transfer with chemical reaction is of considerable importance in chemical and hydrometallurgical industries. In processes such as drying, evaporation at the surface of a water body, energy transfer in a wet cooling tower and the flow in a desert cooler, heat and mass transfer occur simultaneously. Possible applications of this type of flow can be found in many industries. For example, in the power industry, among the methods of generating electric power is one in which electrical energy is extracted directly from a moving conducting fluid.

In many engineering areas processes occur at high temperatures, so knowledge of radiation heat transfer beside the convective heat transfer play a very important role that cannot be neglected. Nuclear power plants, gas turbines and the various propulsion devices for aircraft, missiles, satellites and space vehicles are few examples of such engineering areas.

In view of the above studies, we consider the first order chemical reaction and thermal radiation effects on unsteady MHD flow of an incompressible viscous fluid mixture past a moving vertical cylinder with heat and mass transfer. The governing dimensionless coupled non-linear partial differential equations are solved numerically using finite difference method (FDM) by developing suitable codes in MATLAB. The fluid velocity, temperature and species concentration profiles have been drawn for time and various flow parameters, and effects of various parameters on velocity, temperature and concentration are studied.

MATHEMATICAL ANALYSIS

Consider a two-dimensional unsteady flow of an incompressible viscous electrically, thermally conducting and radiating fluid mixture past an impulsively started semi-infinite vertical cylinder of radius r_0. Here the *x*-axis is taken along the axis of cylinder in the vertical direction and the radial coordinate r is taken normal to the cylinder. The gravitational acceleration *g* is acting downward. Initially both, the cylinder and the fluid, are stationary and are maintained at the same temperature T'_∞ and the same concentration level C'_∞ for all $t' \le 0$. At time $t' > 0$, the cylinder starts moving in the vertical direction with a uniform velocity u_0. At later time, the surface of cylinder is raised to a uniform temperature T'_w and concentration C'_w. The viscous dissipation is assumed to be neglected in the energy equation. All the fluid properties are assumed to be constant except the influence of the density variation, which induces the buoyancy force. It is further assumed that the interaction of the induced axial magnetic field with the flow is considered to be negligible compared to the interaction of the applied magnetic field B_0, with the flow.



Schematic Representation and Coordinate System of the Problem

There exists a homogeneous first order chemical reaction between the fluid and species concentration. The system is considered to be axis symmetric. The induced current does not change the magnetic field. The magnetic field is constant in a direction perpendicular to the cylinder. The coefficient of electrical conductivity is constant and scalar throughout the fluid. Then under these assumptions and the Boussinesq's approximation, the flow is governed by the following system of equations:

Continuity Equation: $$\frac{\partial(ru)}{\partial x}+\frac{\partial(rv)}{\partial r}=0 \quad (1)$$

Momentum equation: $$\frac{\partial u}{\partial t'}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial r}=g\beta(T'-T'_\infty)+g\beta^*(C'-C'_\infty)+\frac{\upsilon}{r}\frac{\partial}{\partial r}(r\frac{\partial u}{\partial r})-\frac{\sigma B_0^2}{\rho}u \quad (2)$$

Energy equation: $$\frac{\partial T'}{\partial t'}+u\frac{\partial T'}{\partial x}+v\frac{\partial T'}{\partial r}=\frac{\alpha}{r}\frac{\partial}{\partial r}(r\frac{\partial T'}{\partial r})-\frac{1}{\rho C_p}\frac{1}{r}\frac{\partial}{\partial r}(rq_r) \quad (3)$$

Mass diffusion equation: $$\frac{\partial C'}{\partial t'}+u\frac{\partial C'}{\partial x}+v\frac{\partial C'}{\partial r}=\frac{D}{r}\frac{\partial}{\partial r}(r\frac{\partial C'}{\partial r})-k_r(C'-C'_\infty) \quad (4)$$

The initial and boundary conditions are:

$$\left.\begin{array}{l} t'\le 0: u=0, v=0, T'=T_\infty', C'=C_\infty' \text{ for all } x\ge 0 \text{ and } r\ge 0 \\ t'>0: u=u_0, v=0, T'=T_w', C'=C_w' \text{ at } r=r_0 \\ \quad u=0, T'=T_\infty', C'=C_\infty' \text{ at } x=0 \\ \quad u\to 0, T'\to T_\infty', C'\to C_\infty' \text{ as } r\to\infty \end{array}\right\} \quad (5)$$

where x is the spatial coordinate along the cylinder and r is the spatial coordinate normal to the cylinder. u and v denote the velocity components in the x and r directions, respectively. t' is the time, υ is the kinematic viscosity, β is the volumetric coefficient of thermal expansion , β^* is the volumetric coefficient of expansion for concentration, B_0 is the external magnetic field, ρ is the density, σ is the electrical conductivity , α is the thermal diffusivity, g is the acceleration due to gravity, T' is the temperature of the fluid near the cylinder, T'_w is temperature of the cylinder, T'_∞ is temperature in the free stream, C' is the species concentration near the cylinder, C_p is the specific heat at constant pressure, C'_∞ is the species concentration in the free stream, C'_w is the species concentration of the cylinder, D is the chemical molecular diffusivity, q_r is the radiative flux and k_r is the rate constant for first order chemical reaction.

The radiative heat flux term is simplified by using the Rosseland approximation as $$q_r=-\frac{4\sigma_s}{3k_e}\frac{\partial T'^4}{\partial y} \quad (6)$$

where σ_s is the Stefan-Boltzmann constant and k_e the mean absorption coefficient. It should be noted that by using the Rosseland approximation, the present analysis is limited to optically thick fluids. If temperature differences within the flow are sufficiently small, then Equation (6) can be linearized by expanding T'^4 into the Taylor series about T'_∞ which after neglecting higher order terms takes the form $$T'^4\cong 4T'^3_\infty T'-3T'^4_\infty \quad (7)$$

In view of Equations (6) and (7), Equation (3) reduces to

$$\frac{\partial T'}{\partial t'}+u\frac{\partial T'}{\partial x}+v\frac{\partial T'}{\partial r}=\frac{\alpha}{r}\frac{\partial}{\partial r}(r\frac{\partial T'}{\partial r})+\frac{16\sigma_s T_\infty'^3}{3k_e\rho C_p}\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial T'}{\partial r}) \quad (8)$$

In order to write the governing equations and the boundary conditions in dimensionless form, the following non-dimensional quantities are introduced.

$$\left.\begin{array}{l} X=\frac{x\upsilon}{u_0 r_0^2}, R=\frac{r}{r_0}, t=\frac{t'\nu}{r_0^2}, U=\frac{u}{u_0}, V=\frac{vr_0}{\upsilon}, Gr=\frac{g\beta r_0^2(T_w'-T_\infty')}{\upsilon u_0}, Gm=\frac{g\beta^* r_0^2(C_w'-C_\infty')}{\upsilon u_0}, \\ N=\frac{K_e K}{4\sigma_s T_\infty'^3}, M=\frac{\sigma B_0^2 u_0^2}{\rho v}, T=\frac{T'-T_\infty'}{T_w'-T_\infty'}, C=\frac{C'-C_\infty'}{C_w'-C_\infty'}, \Pr=\frac{\upsilon}{\alpha}, Sc=\frac{\upsilon}{D}, K_r=\frac{k_r r_0^2}{\upsilon}. \end{array}\right\}$$

(9)

We get the following non-dimensional form

$$\frac{\partial(RU)}{\partial X}+\frac{\partial(RV)}{\partial R}=0 \quad (10)$$

$$\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial R}=GrT+GmC+\frac{1}{R}\frac{\partial}{\partial R}(R\frac{\partial U}{\partial R})-MU \quad (11)$$

$$\frac{\partial T}{\partial t}+U\frac{\partial T}{\partial X}+V\frac{\partial T}{\partial R}=\frac{1}{\Pr}(1+\frac{4}{3N})\frac{1}{R}\frac{\partial}{\partial R}(R\frac{\partial T}{\partial R}) \quad (12)$$

$$\frac{\partial C}{\partial t}+U\frac{\partial C}{\partial X}+V\frac{\partial C}{\partial R}=\frac{1}{Sc}\frac{1}{R}\frac{\partial C}{\partial R}+\frac{1}{Sc}\frac{\partial^2 C}{\partial R^2}-K_r C \quad (13)$$

The corresponding initial and boundary conditions are:

$$\left.\begin{array}{l} t\le 0: U=0, V=0, T=0, C=0 \\ t>0: U=1, V=0, T=1, C=1 \text{ at } R=1 \\ U=0, T=0, C=0 \text{ at } X=0 \\ U\to 1, T\to 0, C\to 0 \text{ as } R\to\infty \end{array}\right\} \quad (14)$$

Where, Gr is the thermal Grashof number, Gm is the mass Grashof number, M is the magnetic parameter, Pr is the prandtl number, N is the thermal radiation parameter, Sc is the Schmidt number and K_r is the chemical reaction parameter.

SOLUTION OF THE PROBLEM

The dimensionless governing differential equations (11) - (13) subject to the initial and boundary conditions (14) are reduced to a system of difference equations using the following finite difference scheme, and then the system of difference equations is solved numerically by an iterative method. The scheme for a variable U is given by

$$\frac{\partial U}{\partial X}=\frac{U_{i+1,j,k}-U_{i,j,k}}{\Delta X},\frac{\partial U}{\partial R}=\frac{U_{i,j+1,k}-U_{i,j,k}}{\Delta R},\frac{\partial U}{\partial t}=\frac{U_{i,j,k+1}-U_{i,j,k}}{\Delta t},\frac{\partial^2 U}{\partial X^2}=\frac{U_{i+1,j,k}-2U_{i,j,k}+U_{i-1,j,k}}{(\Delta X)^2}$$

Equations (9) - (11) are of the form given below

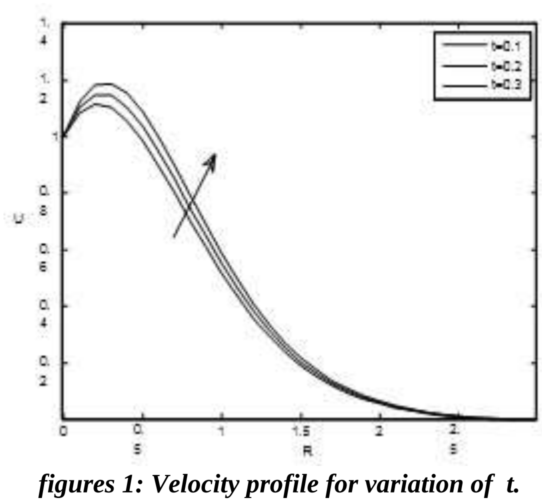
$$\frac{U_{i,j,k+1}-U_{i,j,k}}{\Delta t}+U(i,j,k)\frac{U_{i+1,j,k}-U_{i,j,k}}{\Delta X}+V(i,j,k)\frac{U_{i,j+1,k}-U_{i,j,k}}{\Delta R}=GrT(i,j,k)+GrC(i,j,k)$$
$$+\frac{1}{R}\frac{U_{i,j+1,k}-U_{i,j,k}}{\Delta R}+\frac{U_{i,j+1,k}-2U_{i,j,k}+U_{i,j-1,k}}{(\Delta R)^2}-MU(i,j,k)$$

$$\frac{T_{i,j,k+1}-T_{i,j,k}}{\Delta t}+U(i,j,k)\frac{T_{i+1,j,k}-T_{i,j,k}}{\Delta X}+V(i,j,k)\frac{T_{i,j+1,k}-T_{i,j,k}}{\Delta R}=\frac{1}{\Pr}(1+\frac{4}{3N})(\frac{1}{R}\frac{T_{i,j+1,k}-T_{i,j,k}}{\Delta R}$$
$$+\frac{T_{i,j+1,k}-2T_{i,j,k}+T_{i,j-1,k}}{(\Delta R)^2})$$

$$\frac{C_{i,j,k+1}-C_{i,j,k}}{\Delta t}+U(i,j,k)\frac{C_{i+1,j,k}-C_{i,j,k}}{\Delta X}+V(i,j,k)\frac{C_{i,j+1,k}-C_{i,j,k}}{\Delta R}=\frac{1}{Sc}\frac{1}{R}\frac{C_{i,j+1,k}-C_{i,j,k}}{\Delta R}$$
$$+\frac{1}{Sc}\frac{U_{i,j+1,k}-2U_{i,j,k}+U_{i,j-1,k}}{(\Delta R)^2}-K_rC(i,j,k)$$

RESULTS AND DISCUSSION

Chemical reaction(K_r) and thermal radiation(N) on the velocity(U), temperature(T) and concentration(C) profiles are studied for various values of the parameters, which describe the flow characteristics and the results are reported in terms of graphs. Numerical calculations have been carried out for different values of t, K_r, N and for fixed values of Pr, Sc, Gr, Gm and M. The values are taken for computation t = 0.2, Gr = 1, Gm = 1, M = 1, Pr =0.71, Sc = 0.66, N = 10, K_r = 1.



figures 1: Velocity profile for variation of t.

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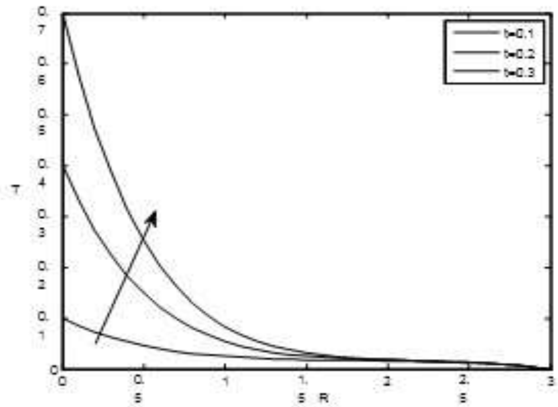
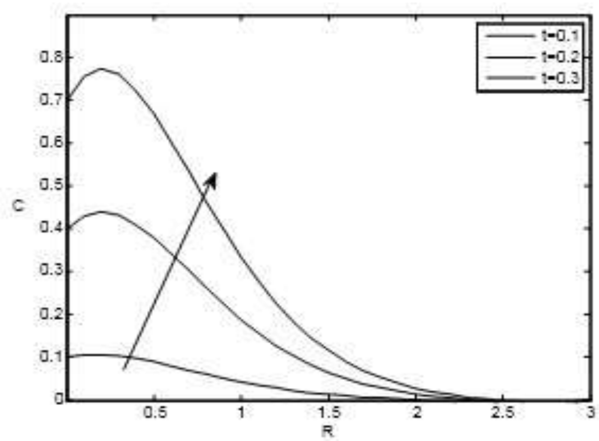
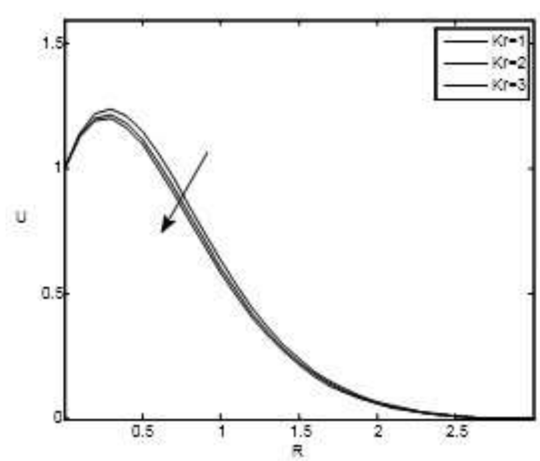


figures 2: Temperature profile for variation of t.



figures 3: Concentration profile for variation of t.

In the above figures 1, 2, 3 show the velocity, temperature and concentration profiles respectively against R for several values of the time t. It is observed that an increase in t leads to an increase in the values of velocity, temperature and concentration.



Figures 4: Velocity profile for variation of k_r

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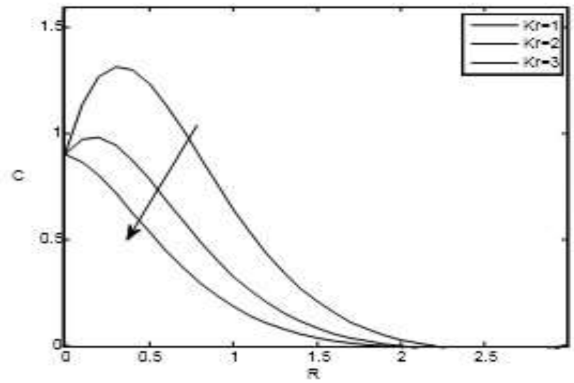
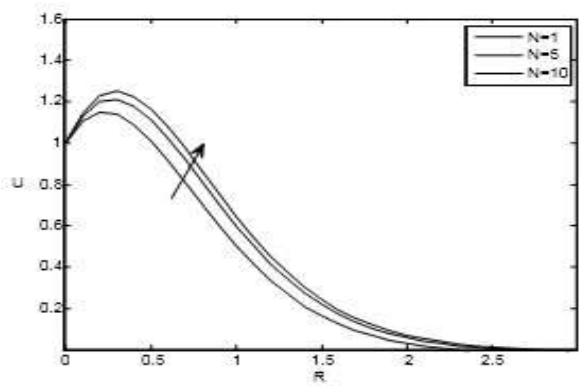


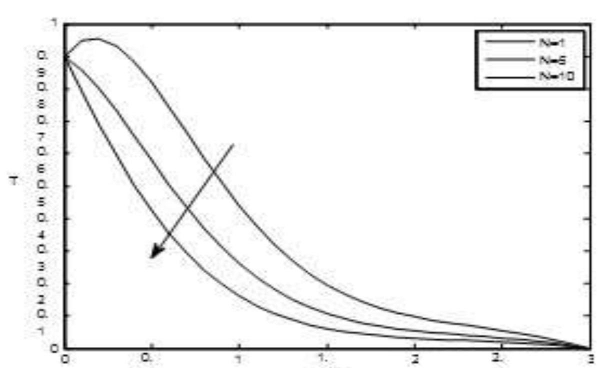
figures 5: Concentration profile for variation of k_r

The effect of Chemical reaction (K_r) on velocity and concentration profile is shown in figures 4 and 5. It is observed that the velocity and concentration decreases with the increasing values of K_r. Due to increase in values of K_r, the concentration of fluid particles near the surface of the cylinder, which results in decreasing the effect of mass buoyancy forces and thus decrease the fluid velocity.

Due to increase in K_r, the constituents from higher concentration zone (adherent to the surface) moves towards the species in lower concentration zone(free stream) results of which decreases the concentration boundary layer thickness, thus decreasing the values of concentration.



figures 6: Velocity profile for variation of N.



figures 7: Temperature profile for variation of N.

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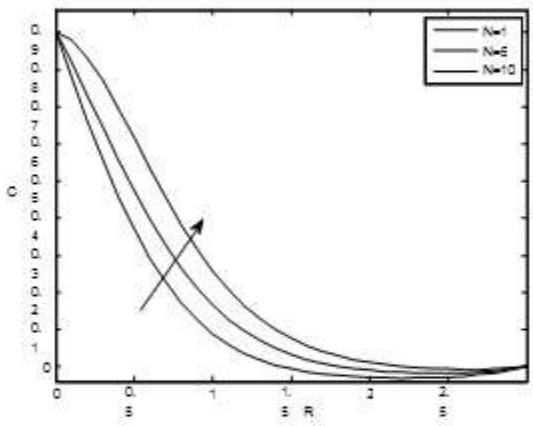

figures 8: Concentration profile for variation of N.

The effect of thermal radiation (N) on velocity, temperature and concentration profiles is depicted in figures 6, 7 and 8. It is observed that the velocity and concentration increases with the increasing values of N because the velocity boundary layer thickness increases and the thickness of the concentration boundary layer also increase. Which means the velocity and concentration gradients at the surface increase enhances the fluid velocity and concentration. and temperature decreases.

CONCLUSIONS

A numerical study has been carried out for the flow of an incompressible viscous fluid past a moving vertical cylinder with the effects of chemical reaction and thermal radiation. The governing dimensionless coupled linear partial differential equations are solved numerically using finite difference method (FDM) by developing suitable codes in MATLAB. The results are obtained for time and different values of chemical reaction parameter (K_r) and thermal radiation parameter (N). The following conclusions are drawn:

(**i)** Velocity, temperature and concentration increases with increase in time.

(ii) Velocity and concentration decreases with increase in chemical reaction parameter (K_r).

(iii) Velocity and concentration increases with increase in thermal radiation parameter (N) but decrease temperature.

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