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ON THE CONTROLLABILITY OF FUZZY DIFFERENCE CONTROL SYSTEMS

T. Srinivasa Rao, G. Suresh Kumar*, Ch. Vasavi and B. V. Appa Rao
Department of Mathematics,
Koneru Lakshmaiah Education Foundation
Vaddeswaram, Guntur Dt, Andhra Pradesh, India

ABSTRACT

In this paper, we study the fuzzy logic system from the aspect of fuzzy difference equations. One of its features, controllability was studied. We obtain sufficient condition for the existence of a fuzzy control which transfers the system from initial state to a desired state in a finite time.

Key words: Fuzzy Dynamical Difference System, Controllability, A-Level Sets.

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1. INTRODUCTION

The theory of difference equations plays an important role in areas such as digital control, digital filter design and image processing with the emergence of digital signal processing technology. The importance of control theory in applied mathematics and its occurrence in several fields such as mechanics, electromagnetic theory, thermodynamics and artificial satellites are well known. Controllability is one of the fundamental concepts in modern mathematical control theory. It means that, it is possible to steer a dynamical system from an arbitrary initial state to an arbitrary final state using suitable controls. The problem of controllability for a system of ordinary differential equations was studied in [1] and for matrix Lyapunov systems by Murty et al. in [2].

Fuzzy logic systems are very useful in helping to establish an intelligent control theory. Fuzzy control usually decomposes a complex system into several subsystems and uses a simple control according to the understanding of the human expert's system. Ding and Kandel [3] provide a way to combine differential equations with fuzzy sets to form fuzzy logic systems called fuzzy dynamical systems which is the new approach to intelligent control and also studied controllability of this system. Murty et al. [4, 5] extended this approach to fuzzy dynamical matrix Lyapunov systems and obtained sufficient conditions for controllability. In [6], the authors studied observability of fuzzy difference control systems. In this paper, we provide a way to incorporate difference equations with fuzzy sets to form a new fuzzy logic system called fuzzy difference system which can be regarded as a new approach to intelligent control. We consider the linear difference control system of the form:

$$x(n+1) = A(n)x(n) + B(n)u(n),\ \ x(0) = x_0; \tag{1.1}$$

$$y(n) = c(n)x(n) + D(n)x(n), \tag{1.2}$$

where A(n) is a non-singular matrix and B, C, D are matrix functions of n on $J = [0,T] \cap N$, $T \in N = \{0,1,2,....\}$. If the inputs $u(n)$ are crisp, then it is the classical discrete control system. Here, we take the input $u(n)$ is a fuzzy set on R^n. We prove that the state of the system (1.1), (1.2) is also a fuzzy set and provide a sufficient condition for the controllability of this system.

This paper is organized as follows. In Section 2, we present some basic definitions and results related to fuzzy sets and fuzzy difference equations. In Section 3, we generate a fuzzy difference control system using the deterministic control system with fuzzy inputs and outputs. In section 4, we present a sufficient condition for the controllability of the system and illustrate the result with suitable example.

2. PRELIMINARIES

In this section, we present some results on fuzzy sets and fuzzy difference equations which are useful for later discussion. Let $P_k(R^m)$ denotes the space of all nonempty compact convex subsets of R^m. For any $P_k(R^m)$, the Hausdorff metric is given by

$$d_H(A,B) = \max\left\{ \sup_{a\in A} \inf_{b\in B} \| a-b \|, \sup_{b\in B} \inf_{a\in A} \| a-b \| \right\},\quad \left(P_k(R^m), d_H\right)$$ is a complete and separable metric space .

Let S_F the set of all selections of $F(.)$ that belong to the space of summable functions

$$S_{R^m}(J) = \left\{ f \in R^m / \sum_J \| f \| < \infty \right\},\ \text{i.e.,}$$

$$S_F = \{ f(.) \in S_{R^m}(J)\ \ / f(t) \in F(t)\ \ a.e. \}$$

We define the summable function for multi-valued functions as follows

$$\sum_J F(n) = \left\{ \sum_J f(n), f(.) \in S_F \right\}$$

Define $E^m = \{u : R^m \to [0,\ 1]\}$, where u is normal, fuzzy convex, upper semi continuous and support $[u]^0$ is compact.

For $0 < \alpha \le 1$, , denote $[u]^\alpha = \{x \in R^m / u(x) \ge \alpha\}$, then the α-level set $[u]^\alpha \in P_k(R^m)$ for all $0 < \alpha \le 1$. Define $D : E^m \times E^m \to [0,\infty)$ by the equation $D(u,v) = \sup_{0\le\alpha\le1} d_H\left([u]^\alpha, [v]^\alpha\right)$, where d_H is the Hausdorff metric defined in $P_k(R^m)$. The addition and scalar multiplication for any two fuzzy sets $u, v \in E^m$, and for any scalar $\lambda \in R$.

$[u+v]^\alpha = [u]^\alpha + [v]^\alpha, [\lambda u]^\alpha = \lambda[u]^\alpha$, for $0 \le \alpha \le 1$. Then (E^m, D) is a complete metric space. For properties on D, we refer to [8, 9].

Definition 2.1. A fuzzy set valued function $F : J \to E^m$ is called summably bounded, if it is measurable and there exists a summable function h such that $\| y \| \le h(n)$ for all $y \in F_0(n)$.

A fuzzy set valued function $F : J \to E^m$ is called summably bounded if F_α is summable bounded for all α in [0, 1].

Definition 2.2. Let $F : J \to E^m$ be a fuzzy summably bounded mapping. The fuzzy summation of F over J denoted by $\sum_J F(n)$, is defined level wise by $\left[\sum_J F(n)\right]^\alpha = \sum_J F_\alpha(n)$.

This fuzzy summation is well defined and we will show that $\sum_J F(n) \in E^m$.

Theorem 2.1. If the fuzzy set valued mapping $F : J \to E^m$ is summably bounded, then $\sum_J F \in E^m$.

Proof. The proof is similar to proof of Theorem 3.1 [7]

Theorem 2.2. If $F : J \to E^m$ is strongly measurable and summably bounded, then F is summable.

Proof. The proof is similar to the proof in [7]

Theorem 2.3. Let $F : J \to E^m$ be two fuzzy set valued functions which are fuzzy summable and $\lambda \in R$.. Then

(i) $\sum(F+G) = \sum F + \sum G$

(ii) $\sum \lambda F = \lambda \sum F$

(iii) $D(F,G)$ is summable.

(iv) $D(\sum F, \sum G) = \sum D(F,G)$

Proof: (i) Let $\alpha \in [0, 1]$. Since F_α and G_α are compact convex valued, it follows that summation $\sum F_\alpha$ and $\sum G_\alpha$ are in S_F. Hence

$$\sum(F+G)_\alpha = \sum F_\alpha + \sum G_\alpha = \sum F_\alpha + \sum G_\alpha.$$

(ii) Let $\lambda \in R$ and $\alpha \in [0,1]$. Since F is compact convex closed, it follows that $\sum(\lambda F_\alpha)$ is in S_F. Hence

$$\sum(\lambda F)_\alpha = \sum \lambda F_\alpha = \lambda \sum F_\alpha$$

(iii) Let $\{ f_n^\alpha / n = 1, 2, 3, ...\}, \{ g_n^\alpha / n = 1, 2, 3, ...\}$ be a casting representation for F_α and G_α . By the definition of D,

$D(F(n),G(n)) = \sup_{k\geq 1} d_H\left(F_{\alpha_k}(n), G_{\alpha_k}(n)\right)$, Where $\{\alpha_k / k = 1,2,3,....\}$ is dense in [0, 1], is measurable. Further,

$D(F(n), G(n)) \leq D(F(n), \bar{0}) + D(G(n),\bar{0}) \leq h_1(n) + h_2(n)$ where h_1 and h_2 are summable bounds for F and G respectively. Hence from Theorem 2.2, $D(f, g)$ is summable.

(iv) From the def of d_H, we have $d_H\left(\sum F_\alpha, \sum G_\alpha\right) \leq \sum d_H(F_\alpha, G_\alpha)$, and consequently

$$D\left(\sum F, \sum G\right) \leq \sup_{\alpha\in[0,1]} \sum d_H(F_\alpha, G_\alpha) \leq \sum \sup_{\alpha\in[0,1]} d_H(F_\alpha, G_\alpha) = \sum D(F,G).$$

Let $F : J \times E^m \to E^m$. Consider the fuzzy difference equation

$$\Delta x(n) = f(n, x(n)),\ \mathrm{x}(0) = \mathrm{x}_0. \qquad (2.1)$$

Definition 2.3: A mapping $x : J \to E^m$ is a fuzzy solution to (2.1), if it satisfies the equation

$$x(n) = x_0 + \sum_{k=0}^{k=n-1} F(k, x(k)),\ \text{for all}\ n \in J$$

Let E^n be the fuzzy space based on R^n and A, B, C, D are crisp matrices in which A is non-singular and x, u are fuzzy sets, we call the system

$$x(n+1) = A(n)x(n) + B(n)u(n),\ x(0) = x_0;$$
$$y(n) = c(n)x(n) + D(n)x(n),$$

fuzzy dynamical control system. The solution to this system will be constructed in the next section. The fundamental matrix of the equation

$$x(n+1) = A(n)x(n),\ \text{ for } n \geq 0, \qquad (2.2)$$

is denoted by $Y(n) = \begin{cases} A(n-1)(n-2)(n-3)......A(1)A(0);\ \mathrm{n} > 0 \\ I_m;\ \mathrm{n} = 0 \end{cases}$ and the solution of (2.2) with $x(0) = x_0$ is $x(n) = Y(n)x_0$.

3. FORMATION OF FUZZY DIFFERENCE CONTROL SYSTEM

In this section we show that the deterministic control system (1.1) and (1.2) with fuzzy inputs $u(n)$ determines a fuzzy difference control system.

For $0 < \alpha \leq 1$, let $[u(n)]^\alpha$ be the α-level set of $u(n)$. Consider difference inclusions

$$x_\alpha(n+1) \in A(n)x_\alpha(n) + B(n)[u(n)]^\alpha,\ x(0) = x_0,\ n \in J \qquad (3.1)$$

Let x^α be the solution set of inclusion (3.1).

Claim (i) $[x(n)]^{\alpha} \in P_k(\mathbf{R}^{\mathrm{m}})$ for all $n \in J$. First we prove that x^{α} is non-empty, compact and convex in the set of all functions from J to R^m, Since $[u(n)]^{\alpha}$ has measurable selection, we have x^{α} non-empty. Let $K_1 = \max_{n\in J} \| Y(n) \|$, $\mathrm{K}_2 = \max_{n\in J} \| Y^{-1}(n) \|$,

$$M = \max_{n\in J} \max \{\| u(n) \|, u(n) \in | u(n) |^{\alpha}\} \quad \mathrm{N} = \max_{n\in J} \| B(n) \|,$$

For any $x \in X^{\alpha}$, there is a selection $u(n) \in [u(n)]^{\alpha}$ such that

$$x(n) = Y(n)x_0 + \sum_{k=0}^{n-1} Y(n)Y^{-1}(k+1)B(k)u(k)$$

It implies that

$$\| x(n) \| \le \| Y(n)x_0 \| + \| \sum_{k=0}^{n-1} Y(n)Y^{-1}(k+1)B(k)u(k) \| \le K_1 \| x_0 \| + LK_1K_2MN$$

.

Thus x^{α} is bounded. To prove x^{α} is compact, it is sufficient to prove that it is closed. Let $\{x_m\} \in x^{\alpha}$ and $\{x_m\} \to x.$. For each $\{x_m\} \in x^{\alpha}$, there is a $u_m \in [u(n)]^{\alpha}$ such that

$$x_m(n) = Y(n)x_0 + \sum_{k=0}^{n-1} Y(n)Y^{-1}(k+1)B(k)u_m(k),$$

Since $u_m \in [u(n)]^{\alpha}$ is closed, then there exists a sub sequence $\{u_{m_j}\}$ of $\{\mathrm{u}_{\mathrm{m}}\}$ converging weakly to $u \in [u(n)]^{\alpha}$'. From Mazur's theorem, there exists a sequence of numbers $\lambda_j \succ 0,\ \sum \lambda_{\mathrm{j}} = 1$ such that $\sum \lambda_j u_{mj}$ converges strongly to u. Thus we have

$$\sum \lambda_j x_{mj}(n) = Y(n)x_0 + \sum_{k=0}^{n-1} Y(n)Y^{-1}(k+1)B(k) \sum \lambda_j u_{mj}(k)$$

Taking the limit as $j \to \infty,$ on both sides, we have

$$x(n) = Y(n)x_0 + \sum_{k=0}^{n-1} Y(n)Y^{-1}(k+1)B(k)u(k)$$

Thus $x(n) \in x^{\alpha}$, for all $n \in J$. Hence x^{α} is closed. Now we prove x^{α} is convex. .

Let $x_1, x_2 \in x^{\alpha}$, then there exists $u_1, u_2 \in [u(n)]^{\alpha}$ such that

$$x_1(n+1) = A(n)x_1(n) + B(n)u_1(n),\ x_2(n+1) = A(n)x_2(n) + B(n)u_2(n),$$

Let $x = \lambda x_1 + (1-\lambda)x_2,\ \ 0 \le \lambda \le 1$, then

$$\begin{aligned} x(n+1) &= \lambda x_1(n+1) + (1-\lambda)x_2(n+1) \\ &= \lambda\big(A(n)x_1(n) + B(n)u_1(n)\big) + (1-\lambda)\big(A(n)x_2(n) + B(n)u_2(n)\big) \\ &= A(n)(\lambda x_1(n) + (1-\lambda)x_2(n)) + B(n)(\lambda u_1(n) + (1-\lambda)u_2(n)). \end{aligned}$$

Since $[u(n)]^{\alpha}$ is convex, $\lambda u_1(n) + (1-\lambda)u_2(n) \in [u(n)]^{\alpha}$, we have

$x(n+1) = A(n)x(n) + B(n)[u(n)]^{\alpha}$,

Hence $x \in x^{\alpha}$. Therefore x^{α} is convex. Therefore, x $^{\alpha}$ is nonempty, compact and convex in J. Thus, $[x(n)]^{\alpha}$ is compact and convex for every n in J. Thus, $[x(n)]^{\alpha} \in P_k(R^n)$, for every $n \in J$.

Claim (ii) $[x(n)]^{\alpha_2} \subset [x(n)]^{\alpha_1}$, for all $0 \le \alpha_1 \le \alpha_2$. Let $0 \le \alpha_1 \le \alpha_2$. Since u is a fuzzy set, $[u(n)]^{\alpha_2} \subset [u(n)]^{\alpha_1}$, *then* $S_{[u(n)]^{\alpha_2}} \subset S_{[u(n)]^{\alpha_1}}$ and the following inclusion

$$x_{\alpha_2}(n+1) \in A(n)x_{\alpha_2}(n) + B(n)[u(n)]^{\alpha_2} \subset A(n)x_{\alpha_1}(n) + B(n)[u(n)]^{\alpha_1}$$

It implies that

$$x_{\alpha 2}(n) \in Y(n)x_0 + \sum_{k=0}^{n-1} Y(n)Y^{-1}(k+1)B(k)S_{[u(n)]^{\alpha 2}}$$

$$\subset Y(n)x_0 + \sum_{k=0}^{n-1} Y(n)Y^{-1}(k+1)B(k)S_{[u(n)]^{\alpha 1}}.$$

Thus $[x]^{\alpha_2} \subset [x]^{\alpha_1}$, and hence $[x(n)]^{\alpha_2} \subset [x(n)]^{\alpha_1}$, for all $n \in J$.

Claim (iii) If $\{\alpha_r\}$ is a non-decreasing sequence converging to $\alpha > 0$, then $x^{\alpha}(n) = \bigcap_{r \ge 1} x^{\alpha_r}(n)$.

Since u is a fuzzy set, we have

$[u(n)]^{\alpha} = \bigcap_{r \ge 1} [u(n)]^{\alpha_r}$, which implies that $S_{[u(n)]^{\alpha}} = S_{\bigcap_{r\ge1}[u(n)]^{\alpha_r}}$. Thus

$$x_{\alpha}(n+1) \in A(n)x_{\alpha}(n) + B(n)[u(n)]^{\alpha} = A(n)x_{\alpha}(n) + B(n) \bigcap_{r \ge 1} |u(n)|^{\alpha_r}$$

$$\subset A(n)x_{\alpha}(n) + B(n)[u(n)]^{\alpha_r}, \quad r = 1, 2, 3, ...$$

Thus, $x_{\alpha} \subset x_{\alpha_n}$, $r = 1, 2, 3, ...$ which implies that $x_{\alpha} \subset \bigcap_{r \ge 1} x_{\alpha_r}$.

Let x be the solution of the inclusions

$$x_{\alpha_k}(n+1) \in A(n)x_{\alpha_r}(n) + B(n)[u(n)]^{\alpha_r}, \ r \ge 1$$

Then $x(n) \in Y(n)x_0 + \sum_{k=0}^{n-1} Y(n)Y^{-1}(k+1)B(k)S_{[u(n)]^{\alpha_r}}$, it follows that

$$x(n) \in Y(n)x_0 + \sum_{k=0}^{n-1} Y(n)Y^{-1}(k+1)B(k) \bigcap_{r \ge 1} S_{[u(n)]^{\alpha_r}}$$

$$= Y(n)x_0 + \sum_{k=0}^{n-1} Y(n)Y^{-1}(k+1)B(k)S_{[u(n)]^{\alpha_r}} .$$

This implies that $x \in x^{\alpha}$ and $\bigcap_{r \ge 1} x^{\alpha r} \subset x^{\alpha}$.

Hence $x^{\alpha}(n) = \bigcap_{r \ge 1} x^{\alpha r}(n)$ for all $n \in J$.

Hence there exists $x(n) \in E^m$ on J such that $x^\alpha(n)$ is a solution set to the difference inclusions (3.1). Hence the solution (1.1) and (1.2) is a fuzzy difference control system and it can be expressed as

$$X(n+1) = A(n)X(n) + B(n)U(n) = \{x_0\}, n > 0, \qquad (3.2)$$

$$Y(n) = C(n)X(n) + D(n)U(n). \qquad (3.3)$$

The solution of the fuzzy dynamical system (3.2), (3.3) is given by

$$X(n) \in Y(n)x_0 + \sum_{k=n_0}^{n-1} Y(n)y^{-1}(k+1)B(k)U(k) \qquad (3.4)$$

4. CONTROLLABILITY OF FUZZY DYNAMICAL SYSTEMS

In this section, we discuss the concept of controllability of the fuzzy system (3.2), (3.3).

Definition 4.1. The fuzzy system (3.2), (3.3) is said to be completely controllable if for any initial state $x(0) = \{x_0\}$ and any given final state x_f there exists a finite time $n_1 > 0$ and a control $U(n),\ 0 \le n \le n_1$, such that $x(n_1) = n_f$.

Definition 4.2. A set of functions $f_i(n),\ i = 1,2,3....n$ are said to be linearly independent on J if $\sum_{i=1}^{n} c_i f_i(n) = 0$ for all $n \in J$ implies that $c_i = 0, i = 1,2,.....m$. If any one of the constants c_i is nonzero, the functions are said to be linearly dependent on J.

Remark 4.1. A set of vector functions $f_i(n),\ i = 1,2,3....n$ are linearly independent on J. if and only if the determinant, $\det M(0,L) \ne 0$ where

$$M(0,L) = \sum_{k=0}^{k=L-1} F(k)F^*(k)$$

and $F(n)$ is the matrix whose rows are $f_i(n),\ i = 1,2,3....n$, and F^* (n) is the transpose of F (n).

Lemma 4.1. If F is fuzzy set, then $\sum_{k=0}^{k=L-1} kF = \frac{L(L-1)}{2}F.$

Proof: Let $[F]^\alpha$ be the α-level set of F, since

$$\left[\sum_{k=0}^{k=L-1} kF\right]^\alpha = \sum_{k=0}^{L-1} k[F]_\alpha = \frac{L(l-1)}{2}F_\alpha$$

From the definition of fuzzy set, we have $\sum_{k=0}^{k=L-1} kF = \frac{L(L-1)}{2}F.$

Lemma 4.2. Let P, Q be two fuzzy sets and let F (k) be a function defined on J, satisfying $\sum_{k=0}^{k=L-1} F(k)p = \sum_{k=0}^{L-1} F(K)Q$, then $P = Q$.

Proof: For each α-level, if $P = Q.$, we have

$$\sum_{k=0}^{k=L-1} F(k)P = \left[\sum_{k=0}^{L-1} F(k)P\right]^{\alpha} = \left[\sum_{k=0}^{L-1} F(k)P\right]^{\alpha} = \sum_{k=0}^{L-1} F(k)[Q]^{\alpha}$$

Suppose that $P \neq Q$, then for $0 \leq \alpha \leq 1$, we have $P \neq Q$. Without loss of generality, we assume that $P, Q \in E^1$. Let $P^{\alpha} = [P_{\min}(\alpha), P_{\max}(\alpha)]$ and $Q^{\alpha} = [Q_{\min}(\alpha), Q_{\max}(\alpha)]$. Then we have either (i) $P_{\min}(\alpha) \neq P_{\max}(\alpha)$

$P_{\min}(\alpha) \neq P_{\max}(\alpha)$ or (ii) $Q_{\min}(\alpha) \neq Q_{\max}(\alpha)$ holds. If (i) holds, then

$$\sum_{k=0}^{k=L-1} F(k)P_{\min}(\alpha) \neq \left[\sum_{k=0}^{L-1} F(k)Q_{\min}(\alpha)\right]$$

If (ii) holds, then $$\sum_{k=0}^{k=L-1} F(k)P_{\max}(\alpha) \neq \left[\sum_{k=0}^{L-1} F(k)Q_{\max}(\alpha)\right]$$

Then in both cases (i) and (ii), we have

$$\sum_{k=0}^{k=L-1} F(k)\left[P_{\min}(\alpha), P_{\max}(\alpha)\right] \neq \left[\sum_{k=0}^{L-1} F(k)\left[Q_{\min}(\alpha), Q_{\max}(\alpha)\right]\right]$$

This implies that $\sum_{k=0}^{k=L-1} F(k)P^{\alpha} \neq \left[\sum_{k=0}^{L-1} F(k)Q^{\alpha}\right]$, which is a contradiction, hence $P = Q$.

Theorem 4.1. The fuzzy system (4) is completely controllable if the symmetric controllability matrix

$$M(0,L) = \sum_{k=0}^{L-1} Y(n)Y^{-1}(k+1)B(k)B^{T}(k)(Y^{-1}(k+1))^{T} Y^{T}(n), \tag{4.1}$$

is non-singular. Furthermore, the fuzzy control U(t) which transfers the state of the system from $x(0)=x_0$ to a fuzzy state $x(n)=x_f$ can be chosen as

$$U(k) = \frac{2}{L(L-1)} B^{-1}(k)Y(k+1)Y^{-1}(n)X_f$$
$$- B^{T}(k)(Y^{-1}(k+1))^{T} Y^{T}(n)M^{-1}(0,L)Y(n)x_0. \tag{4.2}$$

Proof. From Remark 4.1, assume that M (0, L) is non-singular. Therefore $M^{-1}(0, L)$ exists. Multiplying $M^{-1}(0,L)Y(n)x_0$ on both sides of (4.1), we have

$$Y(n)x_0 = \sum_{k=0}^{L-1} Y(n)Y^{-1}(k+1)B(k)B^{T}(k)(Y^{-1}(k+1))^{T} Y^{T}(n)M^{-1}(0,L)Y(n)x_0. \tag{4.3}$$

The problem is to find the control $U(n)$ exist such that

$x(n) = X_f = Y(n)x_0 + \sum_{k=0}^{L-1} Y(n)Y^{-1}(k+1)B(k)U(k)$ From Lemma 4.1, X_f

can be written as

$$X_f = \frac{2}{L(L-1)} \sum_{k=0}^{L-1} Y(n) Y^{-1}(k+1) B(k) B^{-1}(k)(Y(k+1)) Y^{-1}(n) X_f \tag{4.4}$$

Substituting X_f, we have

$$\frac{2}{L(L-1)} \sum_{k=0}^{L-1} Y(n) Y^{-1}(k+1) B(k) B^{-1}(k)(Y(k+1)) Y^{-1}(n) X_f \tag{4.5}$$

$$= Y(n) x_0 + \sum_{k=0}^{L-1} Y(n) Y^{-1}(K+1) B(k) U(k)$$

Combining (4.3) with the above equation, we get

$$\frac{2}{L(L-1)} \sum_{k=0}^{L-1} Y(n) Y^{-1}(k+1) B(k) B^{-1}(k)(Y(k+1)) Y^{-1}(n) X_f$$

$$= \sum_{k=0}^{L-1} Y(n) Y^{-1}(k+1) B(k) B^{T}(k)(Y^{-1}(k+1))^{T} Y^{T}(n) M^{-1}(0,L) Y(n) x_0$$

$$+ \sum_{k=0}^{L-1} Y(n) Y^{-1}(k+1) B(k) U(k).$$

It follows that

$$\sum_{k=0}^{L-1} Y(n) Y^{-1}(k+1) B(k) U(k) =$$

$$\sum_{k=0}^{L-1} Y(n) Y^{-1}(k+1) B(k) \left[\begin{array}{l} \frac{2}{L(L-1)} B^{-1}(k) Y(K+1) Y^{-1}(n) X_f \\ - B^{-1}(k)(Y^{-1}(K+1))^{T} Y^{T}(n) M^{-1}(0,L) Y(n) x_0 \end{array} \right]$$

Comparing on both sides, from Lemma (4.2), we get

$$U(k) = \frac{2}{L(L-1)} B^{-1}(k) Y(k+1) Y^{-1}(n) x_f - B^{T}(k)(Y^{-1}(k+1))^{T} Y^{T}(n) M^{-1}(0,L) Y(n) x_0. \tag{4.6}$$

Remark 4.2

The nonsingularity of the symmetric controllability matrix $M(0,T)$ in Theorem 4.1 is only a sufficient condition but not necessary condition because $U(t)$ is only one of the controls that satisfy (3.3).

CONCLUSIONS

In this paper, we investigated a way to incorporate difference control system with a fuzzy set. Here, a deterministic difference system with fuzzy inputs and fuzzy outputs can generate a fuzzy difference control system (FDCS). Based on this result, we studied controllability properties of the FDCS. First, we provide a sufficient condition for the controllability of the FDCS, that is, for a given fuzzy state; we determine a control which transfers the initial state to the given state in a finite time. The advantage of our approach is that all levels are represented by mathematical formulas. Our future research work will concentrate on the applications of these systems (FDCS) to civil engineering problems.

REFERENCES

[1] Barnett, S, Cameron, R. G; Introduction to Mathematical Control Theory, 2nd edition, Oxford, UK: Clarendon Press, 1985.

[2] Murty, M. S. N., Appa Rao, B.V., and Suresh Kumar, G.; Controllability, Observability and Realizability of matrix Lyapunov systems, Bulletin of the Korean Mathematical Society, **43** (1), 2006, pp.149-159.

[3] Ding, Z and Kandel, A.; On the Controllability of Fuzzy Dynamical Systems (I), *the Journal of Fuzzy Mathematics*, **8**(1), 2000, pp. 203-214.

[4] Murty, M. S. N. and Suresh Kumar, G.; On Controllability and Observability of fuzzy dynamical matrix Lyapunov systems, *Advances in Fuzzy Systems*, **2008**, 2008, pp. 1-16.

[5] Murty, M. S. N., Suresh Kumar, G., Appa Rao, B. V. and Prasad, K. A. S. N. V. On controllability of fuzzy dynamical matrix Lyapunov systems, *Analele Universitatii de vest, Timisoara*, **LI** (2), 2013, pp. 73-86.

[6] Srinivasa rao, T., Suresh Kumar, G., Vasavi, Ch. and Murty, M. S. N, Observability of Fuzzy Difference Control Systems, *International Journal of Chemical Sciences*, **14(**4), 2016, pp. 2516-2526.

[7] Puri, M. L. and Ralescu, D. A.; Fuzzy random variables, *Journal of Matematical Analysis and Applications,* **114**, 986, pp. 409 - 422.

[8] Lakshmikantham, V. and Mohapatra, R. Theory of fuzzy differential equations and inclusions, 1st Edition. London Taylor and Francis, 2003.

[9] Snehal K.Paliwal, Mrs. Shaila P. Kharde, Robinson, Reagan Nnabio and Kpabep, Charity M., A Review On Data Acquisition and Control System for Industrial Automation Application. International Journal of Electronics and Communication Engineering & Technology, 6(7), 2015, pp.26-30.

[10] Arvind N. Nakiya, Mahesh A. Makwana and Ramesh R. Gajera, An Overview of A Continuous Monitoring and Control System For 3-Phase Induction Motor Based On Programmable Logic Controller and Scada Technology, International Journal of Electrical Engineering and Technology (IJEET), Volume 4, Issue 4, July-August (2013), pp. 188-196.

[11] L. Shrimanth Sudheer, Immanuel J., P. Bhaskar, and Parvathi C. S, Volume 4, Issue 2, March – April, 2013, pp. 217-224, ARM7 Microcontroller Based Fuzzy Logic Controller For Liquid Level Control System, International Journal of Electronics and Communication Engineering & Technology (IJECET).

[12] Naga Raju BOYA, Sreelekha KANDE, Vijay Kumar JINDE, Swapna Chintakunta, Mahesh Ungarala And Ramanjappa Thogata, Design and Development of FPGA Based Temperature Measurement And Control System, Volume 4, Issue 4, July-August, 2013, pp. 86-95, International Journal of Electronics and Communication Engineering & Technology (IJECET).

[13] Nudrat Liaqat, Dr. Suhail. A. Qureshi, Liaqat Ali, Muhammad Khalid Liaqat and Muhammad Afraz Liaqat, Designing of An Application Based Control System For Robust and Intelligent 1dof Exoskeleton, Volume 5, Issue 4, April (2014), pp. 11-19, International Journal of Electrical Engineering and Technology (IJEET).

[14] Er. Naser.F.AB.Elmajdub and Prof. Dr. A.K. Bhardwaj, Design Control System of an Aircraft. International Journal of Electrical Engineering & Technology, 7(5), 2016, pp. 89–122

[15] Kaleva, O.; Fuzzy differential equations, *Fuzzy Sets and Systems*, **24**, 1987, pp, 301-317.