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A NOVEL APPROACH FOR DESIGNING HYBRID TURBO CODES USING DOUBLE STAGE INTERLEAVER

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ABSTRACT

Interleaver being an integral component of the communication system. The interleavers tend to avoid the burst errors by splitting the signals and the turbo codes are used to reduce the BER.The paper will describes a new approach to design a new class of turbo codes that would reduce the decoder complexity, such turbo codes will be known as modified turbo codes taking Decoder complexity as the main parameter consideration for the design of modified turbo codes. The author presents the design of two types of modified turbo codes, the one would be called as Low Complexity Hybrid turbo codes (LCHTC) and the other one Improved Low Complexity Hybrid Turbo Codes (ILCHTC).

KEYWORDS: Interleaver, BER, Decoder Complexity, Turbo codes.

INTRODUCTION

The information has started with two big discoveries, the first one is technological and the other is non technological. John Bardeen, along with William Shockley has invented the transistor but the information or the digital communications theory was invented by Shannon in 1948. Today the communication industry has been given the great advantage for these discoveries.The approach to control an error in digital communications has been started with the work of Shannon, and Hamming. Hamming was the first man who has developed forward error controlling schemes but the Shannon has formulated a theoretical limit and the fundamental limits on the efficiency of information and communications systems. They have provided an idea that with the design, such that the errors could be easily corrected, or they can be completely avoided. This was indicated by Shannon's theory that the improvements could be achieved in the performance of information and communication systems, but it did not told about how to achieve these limits.

Difference between the theoretical limits has been stated by Shannon but the practical results have indicated a lot of space for improvement in the information and communication system, which has particularly motivated the researchers from all around to work upon it. The challenges issued by Shannon have met with the rapid development of number of transistors that has been designed on a single silicon chip. A typical example for this is the invention of turbo codes and the logic of iterative decoding that has been implemented in the receiver. The wide development of the silicon IC's, has allowed an iterative decoding to be implemented practically. This practical implementation of the iterative decoding is very challenging due to its complexity in its decoding algorithms.

The improvement for the error correcting capabilities of a code, has helped to improve the quality of any received information in any proportion and also enables the transmission system that can operate in many severe conditions. It plays vital role for many applications such as satellite system for saving of weight (hardware required), in mobile communication it is helpful in antenna size which can be reduced by increasing the quality of signal and similarly for many wireless applications.

Andrew Viterbi in 1960s, was the first man who has presented a famous algorithm for the decoding of convolutional codes, although this algorithm was not practical due to its high storage requirements. But this algorithm has formed the basis of understanding for convolutional codes and also for the iterative decoding scheme for the serially concatenated codes.A work published by Claude and co-authors in 1993 at the ICC conference has revolutionised the field of forward error correction coding (FECC) scheme and modern digital communication systems [Berrou et al.1993]. This has described a method of how to create much more powerful error correction codes using parallel concatenation of convolutional codes. Its main features were the recursive convolutional encoders (RCE) which were interconnected via an interleaver. However, the Global iterative decoding was the one main trick to achieve the near performance of Shannon limit. High gain was also achieved in the BER performance on the existing FEC codes. But the invention of turbo codes took much attention of the researchers to solve the practical issues of turbo. Unfortunately the questions were arising for the accuracy with a highly complex recursive decoding scheme that has even limited understanding of its methods and ofcourse its practical applications. Invention of Block Turbo Codes (BTC) has enabled to close much of the remaining capacity [A. Glavieux, 1994].A known result of information and communication theory is a randomly chosen code of sufficient block length, capable of approaching the value of channel capacity. enough structure that practical decoding is possible. However with standard code structure (such as convolution codes) the complexity in decoding also increases at a much faster rate than the coding gain. For high speed application, the concatenation of standard codes has also proven effective in improving the effective block length while keeping the complexity under control.

TURBO CODES

Turbo codes were first presented at the International Conference on Communications in 1993 by C. Berrou, Glavieux, and Thitimajshima, and since then become a popular area of communications research. Until then, it was widely believed that to achieve near Shannon's bound performance, one would need to implement a decoder with nearly infinite complexity.

Turbo codes play a major role in the channel coding scheme used in communication system like wireless communication. It is due to their exceptional performance that turbo codes are being accepted as 3GPP standard in personal communications. In next era of wireless communications, mainly the 4G applications, there is a need to provide the best QOS (Quality of Service) provisioning. For certain type of transmission like text transmission, the packet loss is intolerable while delay is acceptable. But for real time video, there can be an acceptable degradation in the video, but delay in the system cannot be accepted. Turbo codes are the most adaptable error coding scheme used to adapt to the varying QOS requirement.Turbo codes can be achieved by serial and parallel concatenation of two (or more) codes called the constituent codes using interleaver between them so that data sequence for the two encoders is different [S. Benedetto, 1996 and 1998]. The constituent codes can be either block codes or convolutional codes.

Principle of Turbo Codes

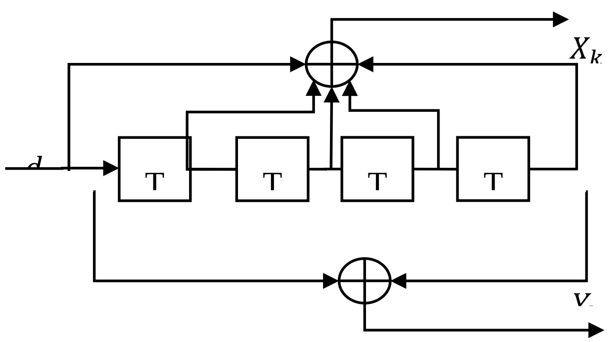
A classical turbo code consists of parallel concatenation of two binary recursive systematic convolutional (RSC) codes and separated by a permutation interleaver [C. Berrou, 1993]. Serial concatenation is also possible. RSC codes are a key component of Turbo Codes. They are based on Linear Feedback Shift-Registers (LFRS) and act as pseudorandom scramblers.

Component codes

The convolutional encoder used in turbo codes for multiple concatenations can be recursive convolutional encoder or non recursive convolutional encoder with the same implementation.

Non Recursive Convolutional codes (NRC)

Non Recursive Convolutional codes [C. Berrou, 1993] are described by a generator polynomial matrix. For an (n, k) convolutional code (there are k input bits and n output bits at every time step), the generator matrix $G(D)$ has k rows and n columns. Nor recursive convolutional code is shown below in the figure.



Non Recursive Convolutional Codes [C. Berrou 1993]

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$$G(D) = \begin{bmatrix} g_{1,1}(D) & \cdots & g_{1,n}(D) \\ \vdots & \ddots & \vdots \\ g_{k,1}(D) & \cdots & g_{k,n}(D) \end{bmatrix} \quad (1)$$

Where, the delay operator is given as:

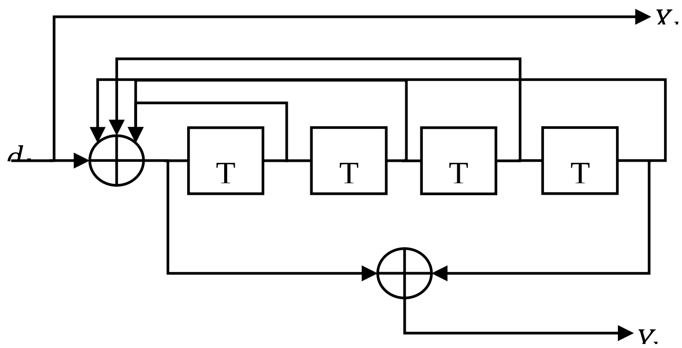
$$d(D) = [d_1(d) \quad d_2(D) \quad \cdots \quad d_k(D)] \quad (2)$$

And the output vector is given as:

$$Y(D) = [y_1(d) \quad y_2(D) \quad \cdots \quad y_n(D)] \quad (3)$$

Recursive Systematic Convolutional Codes (RSC)

In case of Recursive systematic Convolutional codes [C. Berrou, 1993] with k input and n output we still have k input delay registers. In contrast to NRC codes, the input to each register is generated from current input the state of the register according to feedback polynomial.



Recursive Systematic Convolutional Codes

$G(D)$ is given as:

$$G(D) = \begin{bmatrix} 0 & 0 & \cdots & 1 & \frac{g_{1,k+1}(D)}{g_{1,1}(D)} & \cdots & \frac{g_{1,n}(D)}{g_{1,1}(D)} \\ 0 & 0 & \cdots & 1 & \frac{g_{2,k+1}(D)}{g_{2,2}(D)} & \cdots & \frac{g_{2,n}(D)}{g_{2,2}(D)} \\ \vdots & \vdots & \ddots & & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 & \frac{g_{k,k+1}(D)}{g_{k,k}(D)} & \cdots & \frac{g_{k,n}(D)}{g_{k,k}(D)} \end{bmatrix} \quad (4)$$

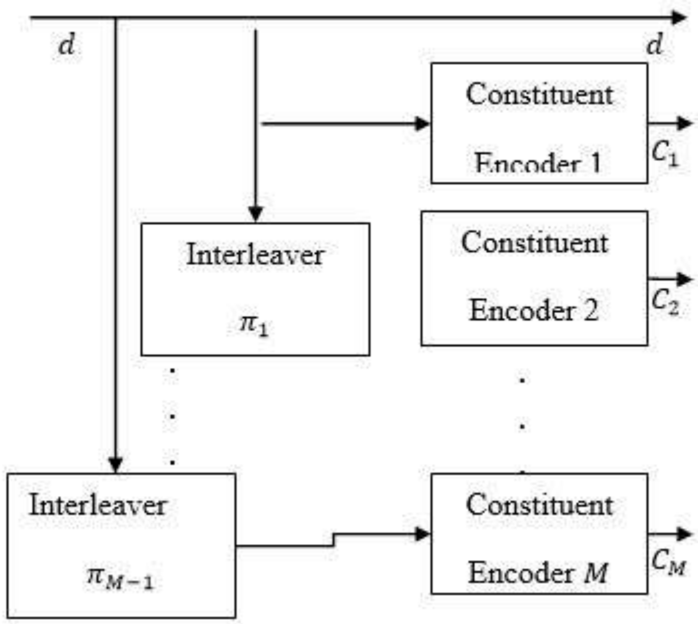
Modified Turbo Codes

Turbo code achieves near Shannon's bound performance.CTC is relatively powerful in terms of error correction, one must be very careful when decoding concatenated codes to achieve near optimum decoding performance. To implement such SISO decoder one would need to implement a decoder with infinite complexity or close. Turbo decoder based on *A Posteriori Probability* (APP) algorithm is computationally complex. If *Maximum A Posteriori* (MAP) algorithm in log domain is used to decode the information bits, the decoder complexity is about 480 Addition Equivalent Operations per Information Bit per Iteration (AEO/IB/I). Decoder complexity of TCC does not reduce even if puncturing is used to adjust the code rate. Also, the decoder complexity increases for wireless channels as Turbo codes require stronger constituent codes and low code rates. These things encourage us to construct low complexity block turbo codes.

It has been shown that concatenated coding schemes, using relatively simple constituent Convolutional and Block codes, can achieve performance close to the theoretical limits and requires low decoding complexity and are termed as *Modified Turbo Codes* (MTC). The MTC solution is more attractive for a wide range of applications. MTC can be implemented using three basic ideas given as follows:

1. The utilization of block codes instead of commonly used non-systematic or systematic convolutional codes.
2. The utilization of soft input soft output decoding. Instead of using hard decisions, the decoder uses the probabilities of the received data to generate soft output which also contain information about the degree of certainty of the output bits.
3. Encoders and decoders works on permuted versions of the same information. This is achieved by using an interleaver.

Modified turbo code consists of concatenation of convolutional code and zigzag code. Zigzag codes are modified form of SPC codes. Since Zigzag codes show better performance than SPC code with slightly more complexity, SPC codes are replaced by Zigzag codes in Modified Turbo Codes. First few component codes of MTC are series combinations of Zigzag code and Convolutional code. Zigzag codes are used in remaining component codes. Use of good codes, like Convolutional Codes, in first few component codes improves error performance of remaining component codes, resulting in overall improvement in error performance. Since Convolutional codes are not used in all component codes, MTC has lower decoding complexity than that of standard Turbo code. BER performance of MTC is close to Turbo code.



Encoder Structure of a General MTC

SISO iterative decoding can be employed for the above shown general MTC. Due to the use of low decoding complexity block code in the constituent encoder, MTC shows less computational complexity than standard TCC. Log MAP decoding of S-state standard turbo codes using two component encoder costs about $30 \times S$ AEO/IB/I. For interleaver of size N trellis length for standard turbo code is N times the number of constituent encoders used. For MTC maximum trellis length is N. Therefore MTC shows less decoding complexity as compared to standard turbo code. Analysis shows that computational complexity of MTC is nearly 50% less than the standard turbo codes. MTC can be classified in two categories as:

1. Low Complexity Hybrid Turbo Code (LCHTC).
2. Improved Low Complexity Hybrid Turbo Code (ILCHTC).

Low Complexity Hybrid Turbo code (LCHTC)

LCHTC encoder uses hybrid concatenation of Zigzag codes and Convolutional codes. First few component codes of LCHTC are series combinations of Zigzag code and Convolutional code. Zigzag codes are used in remaining component codes. Use of good codes, like Convolutional Codes, in first few component codes improves error performance of remaining component codes, resulting in overall improvement in error performance. BER performance of LCHTC is close to Turbo code. However, decoding complexity of LCHTC is considerably lower.

LCHTC Encoder
LCHTC encoder involves parallel concatenation of component codes. First L constituent codes are series concatenation of Zigzag and Convolutional encoders. Resultant codes are Zigzag-Convolutional codes. Then, Zigzag-Convolutional codes are concatenated in parallel with Zigzag codes for performance enhancement. In each constituent encoder Zigzag encoder computes parity bits for each column of information bit array and convolutional encoder compute parity bits for zigzag parity bits. Overall LCHTC encoder uses parallel concatenation of M constituent encoders. Information bits are interleaved before each constituent encoder except for first encoder. Let L be the number of constituent codes using Zigzag-Convolutional encoder. Remaining M-L constituent encoders use Zigzag encoder. The Overall codeword can be given as:

$$C_L = [d, Z_1, Z_2, \dots, Z_M, r_1, r_2, \dots, r_L] \quad (5)$$

Overall code rate can be given as:

$$R = N/[N + 2LK + (M - L)K] \quad (6)$$

Using the value of N from the equation 2.67, equation 2.75 can be simplified as:

$$R = JK/[JK + 2LK + (M - L)K] = J/[J + 2L + (M - L)] \quad (7)$$

LCHTC Decoder
SISO APP decoding is used for decoding of LCHTC codes. Decoding algorithm can be implemented in two steps as follows:

1. Decoding of zig-zag parity bits is performed using RSC decoder.
2. Zig-zag decoder decode information bits taking soft APP out-put of convolutional decoder as input with nosy information bits ($\hat{d}$) as another input.

Improved Low Complexity Hybrid Turbo Codes (ILCHTC)
LCHTC is modified to construct Improved Low Complexity Hybrid Turbo Codes ILCHTC). Information bits are directly encoded by both RSC encoder and zigzag encoder. Since error correcting capability of RSC is better than that of Zigzag code, error convergence in ILCHTC is better than LCHTC.

ILCHTC Encoder
Like LCHTC, ILCHTC uses zigzag code for concatenation with convolutional code. First constituent encoder of ILCHTC uses concatenation of Zigzag encoder and Convolutional encoder. Like LCHTC, ILCHTC uses an information array of size $J \times K$. First constituent encoder uses both Zigzag and Convolutional codes. First, L rows of information array are encoded using a rate $R = 1/2$ RSC code. Figure 2.28 shows the first constituent encoder of ILCHTC with $L = J$.

ILCHTC Decoder
Implementation of decoding algorithm in the first decoder is as follows:

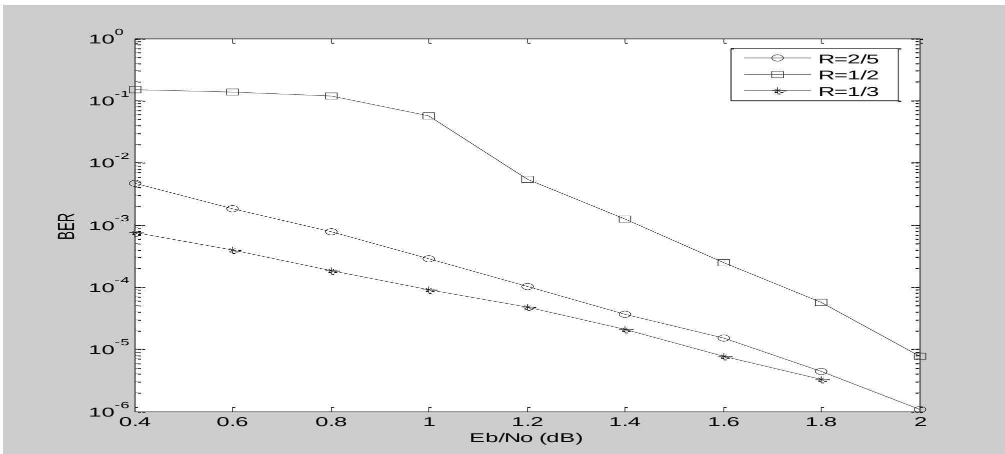
1. Decode each of L rows of the array using *a priori* LLRs as input to SISO Convolutional decoder. Output produced is *a posteriori* LLR of L rows of the information bits array. Decoding can be accomplished by BCJR algorithm using Log-MAP algorithm.
2. Taking output of convolutional decoder as *a priori* LLRs, decode each column of the information array using SISO zigzag decoder.

Add-on toolboxes (collections of special-purpose MATLAB functions, available separately) extend the MATLAB environment to solve particular classes of problems in these application areas. Simulink is used in this research work for deigning and simulating CTC and MTC model.

RESULTS AND DISCUSSION

BER Comparison of Rate $R = 1/2\,, 1/3\ and\ 2/5$ CTC

In communication system bandwidth and data capacity are two important considerations. For rate $R = 1/2$ CTC one information bit produce two code bit, for rate $R = 2/5$ CTC two information bits are coded as five bits and for rate $R = 1/3$ CTC one information bit is coded as three bits. This means as code rate decreases bandwidth required to transmit information signal increases. Comparison of BER performance over AWGN channel is shown in the figure 3.7 for rate $R = 1/2\,, 1/3\, and\ 2/5$ CTC. Bandwidth requirement for rate $R = 1/3$ CTC is more than rate $R = 2/5$ and $R = 1/2$ CTC. Table 3.8 presents comparison of E_b/N_0 to achieve different BER values for $R = 1/3\,, 2/5\ and\ 1/2$ CTC.



BER Comparison for rate $R = 1/2\,, 2/5\ and\ 1/3$ CTC

Simulation result shows that BER performance of rate $R = 1/3$ CTC is best and BER performance of rate $R = 2/5$ CTC is better than rate $R = 1/2$ CTC for low signal to noise ratio. There is a big difference in the BER performance of rate $R = 1/2$ and rate $R = 1/3$ CTC up to $E_b/N_0 = 1\,dB$. For higher value of E_b/N_0 BER performance is nearly same for rate $R = 1/3$ and $R = 1/2$ CTC.

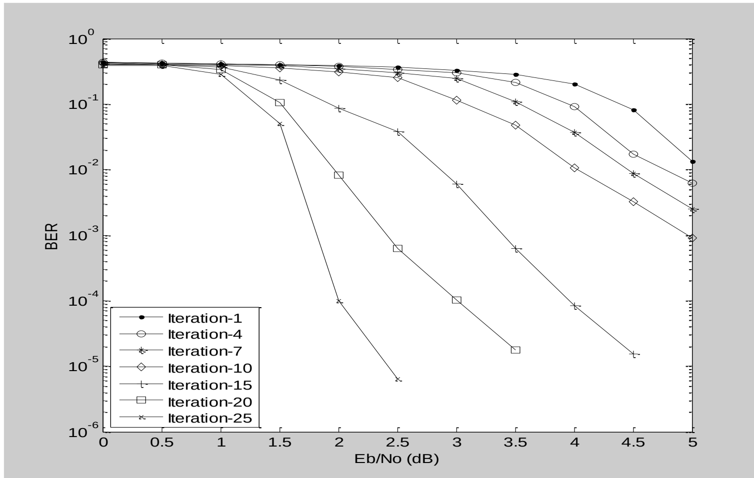
Table 1. E_b/N_0 comparison for rate $R = 1/2\,, 2/5\ and\ 1/3$ CTC

Code Rate R	$E_b/N_0\,(dB) \approx$, for				
	BER $= 10^{-2}$	BER $= 10^{-3}$	BER $= 10^{-4}$	BER $= 10^{-5}$	BER $= 10^{-6}$
$1/2$	1.2	1.5	1.8	2	2.2
$2/5$	0.4	0.8	1.4	1.8	2
$1/3$	0.2	0.4	1.2	1.7	1.9

BER Performance for Rate $R = 1/2$ LCHTC

Simulation result shows that as number of iteration increases *BER* performance of the code improves. Here we have simulated the model up-to 25 iteration. Simulation results show that for first five iteration BER performance of the code is not good. But as number of iteration increases BER performance converges towards a better value. $BER =$

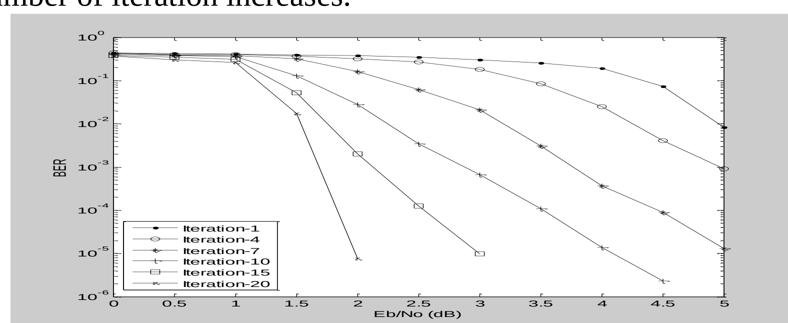
10^{-5} is achieved at $E_b/N_0 = 4.5dB, 3.5dB$ *and* $2.5dB$ for $15^{th}, 20^{th}$ *and* 25^{th} respectively. This shows that there is a gain of $1dB$ for 20^{th} iteration over 15^{th} iteration.



BER Performance for Rate R = 1/2 LCHTC

BER Performance for Rate $R = 1/2$ ILCHTC

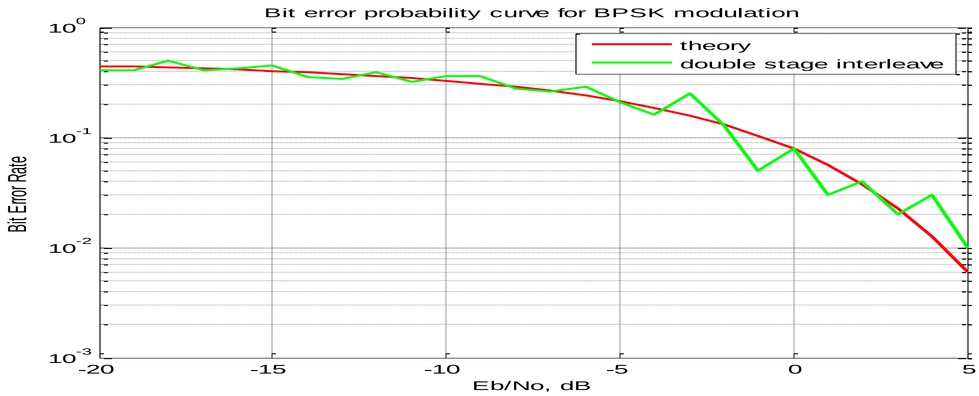
Error rate convergence of convolutional code is better than zigzag codes. so error rate convergence of ILCHTC is better than LCHTC. Simulation results show that *BER* performance of the code improves for successive iteration. At low signal to noise ratio *BER* performance is nearly same for all iteration. But for higher value of signal to noise ratio *BER* converges to a better value as number of iteration increases.



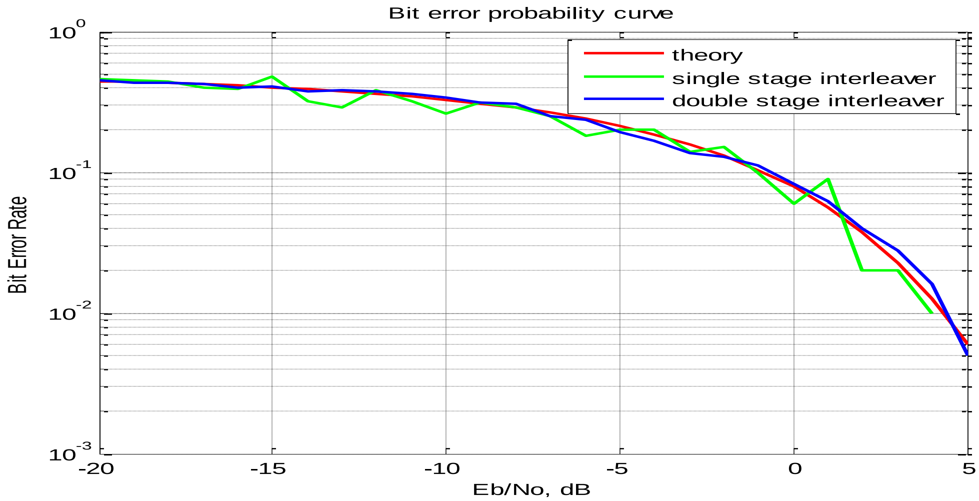
BER Performance for Rate R = 1/2 ILCHTC

Decoder Complexity

For ILCHTC and LCHTC decoder complexity is less due to use of zig-zag code which has lower decoding complexity as compared to CTC. For ILCHTC trellis length depends on No. of column L, encoded by RSC encoder. Rate $R = 1/3$ ILCHTC shows higher decoding complexity than $R = 1/2$ LCHTC, $R = 1/3$ LCHTC and $R = 1/2$ ILCHTC because for first constituent encoder of Rate $R = 1/3$ ILCHTC maximum no of rows $(L = J)$ are encoded by rate $R = 1/2$ RSC encoder. For CTC trellis length is equal to $2N$ where N is No. of information bits.ILCHTC and LCHTC decoder complexity depends on puncturing. Results with two stage interleaver design are shown as with MTC:



BER vs SNR for DOUBLE stage interleaver



Single stage vs Double stage interleaver on the basis of BER

CONCLUSION AND FUTURE SCOPE

It has been investigated from this research work that Decoder complexity is reduced by a factor of nearly 2 for $R = 1/3$ ILCHTC and $R = 1/3$ LCHTC as compared to CTC. For $R = 1/2$ MTC, decoder complexity is reduced by a factor of nearly three as compared to CTC. LCHTC and ILCHTC decoder complexity is changed if puncturing is used to change code rate. Unlike LCHTC and ILCHTC decoder complexity of CTC does not change if puncturing is used to change code rate. Error convergence of CTC is best. ILCHTC and LCHTC require more number of iteration than CTC to get significant BER but overall decoding complexity is less than CTC.

Error convergence of ILCHTC is fast as compared to LCHTC due to use of RSC encoder with zigzag encoder to encode information bit sequence. While for LCHTC information bit sequence is encoded using zigzag encoder and RSC encoder encode zigzag parity sequence. At the receiver side zigzag parity bits are decoded by RSC decoder and information bit sequence is decoded by zigzag decoder for LCHTC decoder. For ILCHTC information bit sequence is decoded by both convolutional decoder and zigzag decoder. Convolution code is more powerful code than zigzag code that's why error convergence is better than the LCHTC encoder.

We have concluded from this research work that decoder complexity is reduced by a factor of nearly two for MTC as compared to CTC with a negligible loss of BER performance. This loss of BER is compensated by reduction of decoder complexity. Memory requirement is much less for decoding MTC as compared to CTC and Two stage interleaver shows better performance over single stage interleaver. Decoder complexity reduction increase speed of decoding and also hardware requirement which may be much beneficial in satellite communication. Such low decoder complexity codes may be explored for the use in digital communication system such as CDMA, OFDM, wireless MIMO technique etc.

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